Further Mathematics - Geometric Interpretation of Solutions of Simultaneous Equation

If planes are given determine their geometric configuration. If matrices are given find the solutions and also sort by their geometric interpretation. There are 5 possible options

1. Three parallel planes.
2. Two parallel planes intersected by a third plane.
3. Triangular prism of planes.
4. 3 planes which intersect at a unique point.
5. 3 planes which intersect along a common line.

1) Solve

$$\begin{aligned} x + 2y + z &= 7 \\ 3x + 5y + 1z &= 16 \\ -x + 2y + 6z &= 11 \end{aligned}$$

2) Find the geometric configuration of the three planes below

$$\begin{aligned} \Pi_1: \quad x + y + 3z &= 4 \\ \Pi_2: \quad 5x + y + 13z &= 4 \\ \Pi_3: \quad x + y + z &= 5 \end{aligned}$$

3) Solve

$$\begin{aligned} 2x + y + z &= 3 \\ x - y - z &= 4 \\ 3x + 3y - 2z &= 3 \end{aligned}$$

4) Solve

$$\begin{aligned} 5x + 2y + z &= 12 \\ -4x + 5y + z &= 9 \\ 3x - 2y + 2z &= 5 \end{aligned}$$

5) Solve

$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \\ 5 & 5 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ -1 \end{pmatrix}$$

6) Two walls are modelled by parallel planes with equations $2x + 3y = 4$ and $2x + 3y = 14$ in a CAD package. Another surface is modelled by the plane $3x + 2y + z = 10$. Determine the geometric configuration of these surfaces.

7) Find the geometric configuration of the three planes below

$$\begin{aligned} \Pi_1: \quad 3x + y - z &= 5 \\ \Pi_2: \quad 6x + 2y - 2z &= 16 \\ \Pi_3: \quad x + y + z &= 2 \end{aligned}$$

8) Solve

$$\begin{pmatrix} 4 & 2 & 1 \\ 7 & 2 & 2 \\ 2 & 1 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

9) Let s be the cost of a starter, m be the cost of a main and d be the cost of a dessert.Three starters, three mains and two desserts cost £41. One starter, one main and one dessert cost £15. Two starters, four mains and 2 desserts cost £46. Form, and solve simultaneous equations for this situation.

10) Solve

$$\begin{pmatrix} 12 & 3 & 5 \\ 3 & 8 & 9 \\ -1 & 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 14 \\ 4 \end{pmatrix}$$

11) Find the geometric configuration of the three planes below

$$\begin{aligned} \Pi_1: \quad 3x + 2y + z &= 5 \\ \Pi_2: \quad 3x + 2\frac{22}{3}y + \frac{13}{32}z &= 9 \\ \Pi_3: \quad x + 6y + z &= 2 \end{aligned}$$

12) Solve

$$\begin{aligned} 3x + y + z &= 0 \\ 5x - y + z &= 0 \\ 8x + 2z &= 0 \end{aligned}$$