FP1 'Keeping Time'

1) Consider the equation $f(x) = 3^x + 3x - 7$.
   a) Show that the equation $f(x) = 0$ has a root α between $x = 1$ and $x = 2$.
   b) Starting with the interval $[1,2]$, use interval bisection twice to find an interval of width 0.25 which contains α.
2) Given that $z = 2 + 4i$,
   a) Find z^2
   b) Find $|z^2|$ and $\arg(z^2)$
   c) Find zz^*
   d) Plot z, z^2 and zz^* on an Argand diagram.
3) Let $A = \begin{pmatrix} 1 & 2 \\ 5 & k \end{pmatrix}$
   a) Find $\det(A)$ and A^{-1} in terms of k.
   b) For what value of k does there not exist an inverse.
   c) Evaluate the determinant when $k = 3$.
4) The parabola C has equation $y^2 = 20x$.
   a) Verify that the point $P(5t^2, 10t)$ is a general point on C.
   b) The point A on C has parameter $t = 4$. The line l passes through A and also passes through the focus of C. Find the gradient of l.
5) Find in the form $p \pm i\sqrt{q}$ the solutions to the quadratic equation $z^2 - 10z + 28 = 0$ and plot these on an Argand diagram.
6) Let $f(x) = x^2 + \frac{5}{2x} - 3x - 1$, $x \neq 0$.
   Taking 0.8 as a first approximation to the root α of the equation $f(x) = 0$ apply the Newton-Raphson process once to obtain a second approximation to α.
7)
   a) Write down a 2×2 matrix that represents an enlargement with centre $(0,0)$ and scale factor 8.
   b) Write down a 2×2 matrix that represents a reflection in the x-axis.
   c) Find the matrix T that represents represents an enlargement with centre $(0,0)$ and scale factor 8, followed by a reflection in the x-axis.
8) Find
   a) $\sum_{r=1}^{n}(6r^2 + 2^r)$
   b) $\sum_{r=4}^{10}(6r^2 + 2^r)$
9) Solve using a matrix method the following simultaneous equations

$$4x - y = 11$$
$$3x + 2y = 0$$

10) The rectangular hyperbola H has Cartesian equation $xy = 4$. The point $P\left(2t, \frac{2}{t}\right)$ lies on H, where $t \neq 0$.

Show that an equation of the normal to H at the point P is

$$ty - t^3x = 2 - 2t^4$$

11) Prove, by induction that, for $n \geq 1$

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$$

12) The quartic equation $z^4 - 5z^3 + 15z^2 - 5z - 26 = 0$ has $z = 2 + 3i$ as one of its roots. Find the other roots.