

The mean value of  $f(x) = 2x^2 + x$  over the interval  $[a, 8]$  is 61. Find  $a$ .

Find the  $2 \times 2$  matrix transformation that represents a rotation of  $90^\circ$  anticlockwise about the origin.

Prove by induction  
$$\sum_{r=1}^n r(r^2 + 3) = \frac{1}{4}n(n+1)(n^2 + n + 6)$$

Find the turning points of the rational function

$$y = \frac{x^2 + 5x + 6}{x^2 + 7x + 6}$$

Use calculus to find the shortest distance between the point  $(1, 3, 2)$  and the line with equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$$

Using Q2 sketch  $y = \frac{x^2 + 5x + 6}{x^2 + 7x + 6}$  fully. Indicate any asymptotes and intersections with the axes.

The cubic  $3x^3 + ax^2 + bx + 4$  has roots  $\alpha, \beta$  and  $\gamma$ . Find

- $\alpha + \gamma + \beta$
- $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

Prove that  $(AB)^{-1} = B^{-1}A^{-1}$

Find the cartesian equation of the loci

$$|z - 2 + 3i| = 3.$$

Sketch the locus of points satisfying

$$|z - 4i| = |z + 3|$$

Draw the graph  $r = 1 - 3 \sin(\theta)$ ,  
 $0 \leq \theta < 2\pi$ .

Use standard results to evaluate  
$$\sum_{r=1}^{20} r^3 + 2r^2 - r$$

Solve the inequality  $\frac{3}{x+2} \leq \frac{1}{x-4}$