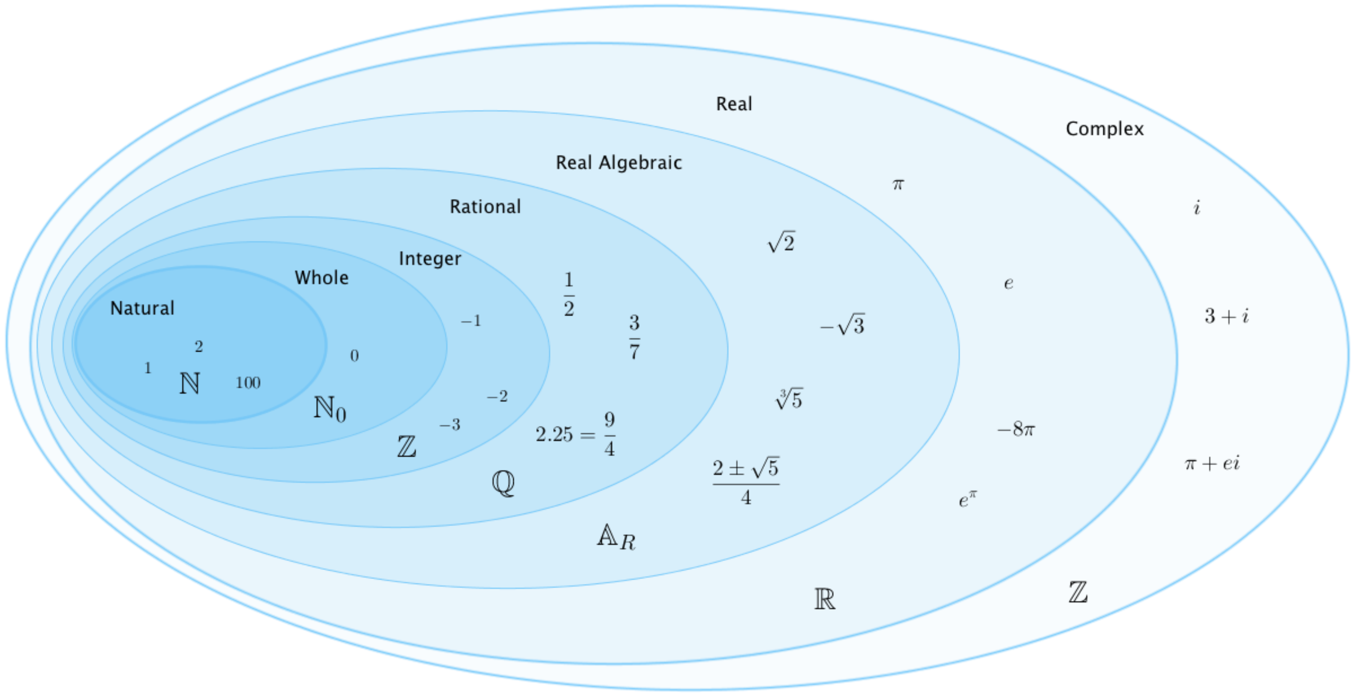
Year 10 Further Maths Taster Session – Number Types and Proof

What types of number are there?



What are complex numbers?

How could you represent complex numbers graphically?

What kind of decimals do you know?

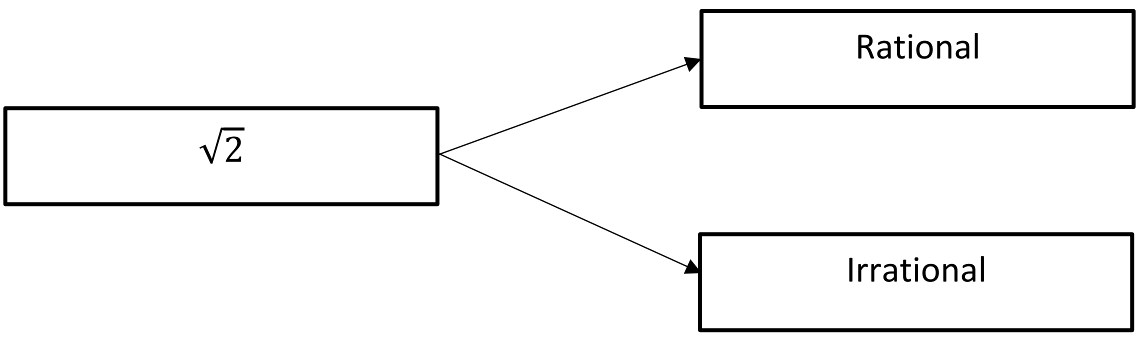
Which of these can be written as a fraction? And how?

An irrational number is a number that cannot be written as $\frac{a}{b}$ with a and b being integers. (Any number that can be written in this way is a rational number) This means that an irrational number cannot be represented as a decimal number which isn't terminating or repeating.

Let us prove that $\sqrt{2}$ is irrational:

Proof:

Any real number is either rational or irrational:



We will use the technique of proof by contradiction to prove that the square root of 2 is irrational.

Let's begin by assuming that $\sqrt{2}$ is rational, if we then get a contradiction we know that $\sqrt{2}$ must be rational.

To this end, assuming $\sqrt{2}$ is rational we can write $\sqrt{2} = \frac{a}{b}$ where the highest common factor of a and b is 1. Squaring both sides we get the following

$$2 = \frac{a^2}{b^2}$$

and so $a^2 = 2b^2$. Now the right hand side is even (it is a multiple of two), so therefore the left hand side is also even, which implies that a is even.

Since a is even we can write $a = 2p$ for some integer p.

Hence,

$$(2p)^2 = 2b^2$$
$$\Rightarrow 4p^2 = 2b^2$$
$$\Rightarrow 2p^2 = b^2$$

Similarly, b is even, and so can be written as $b = 2q$ for some q. This means that we can write $\sqrt{2}$ in the following way:

$$\sqrt{2} = \frac{2p}{2q}$$

But this contradicts our initial assumption that we had written $\sqrt{2}$ as a fraction in its simplest form. Hence our initial supposition must have been incorrect and we can conclude that $\sqrt{2}$ is irrational.

Question: Could an irrational number raised to the power of an irrational number ever be rational?