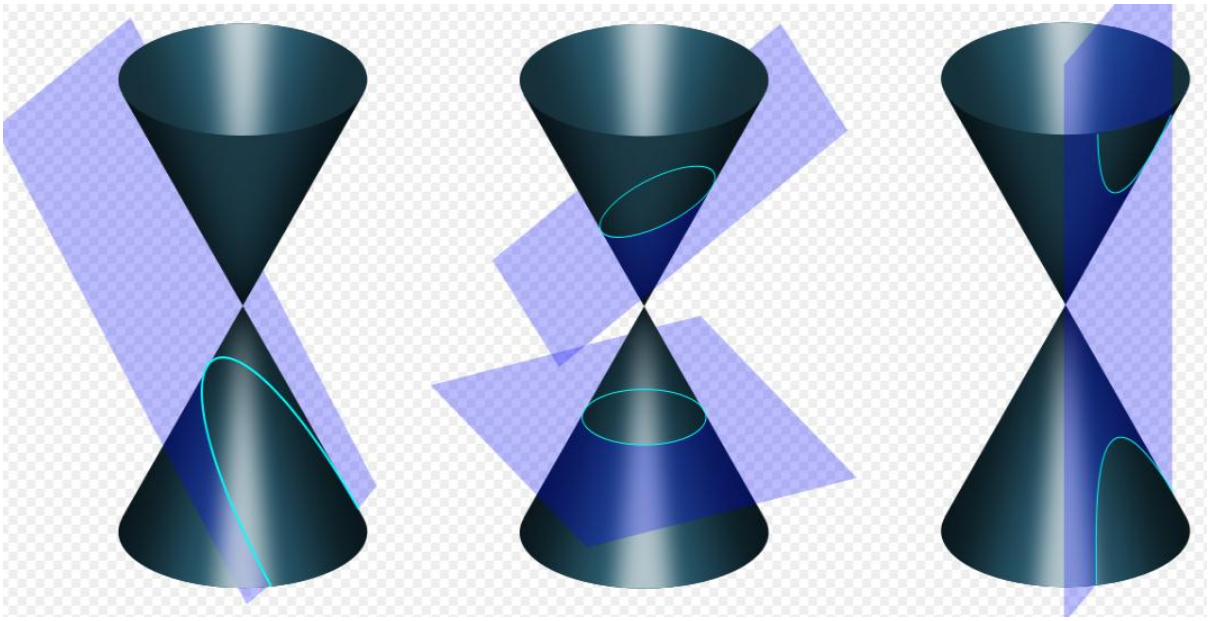
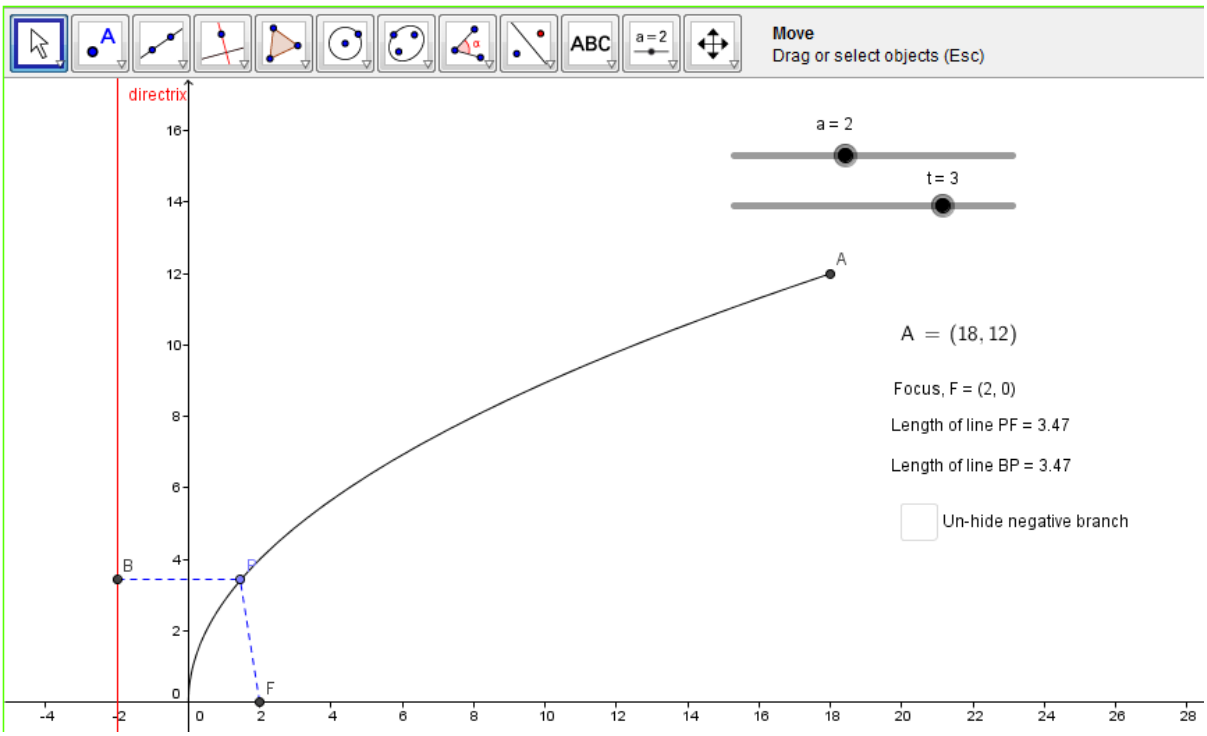
Investigating the Conic Sections

The conic sections are so named because they can all be found by taking sections through a cone, as shown below (image from Wikipedia http://upload.wikimedia.org/wikipedia/commons/d/d3/Conic_sections_with_plane.svg).



In this sheet we are going to investigate two of them using Geogebra applets.

Go to http://bit.ly/181GP1o and you should see an applet that looks like this:



The curve shown here is known as the parabola.

Investigating the Parabola:

1) Select the point P on the parabola and drag it along the parabola. What do you notice about the lengths of the lines PF and BP?

2) Is this true for any value of the parameter a? Hint: Use the top slider to change the value of a.

3) The term “locus” describes a set of points that obey a certain rule. Can you describe the parabola using this word?

4) Set the a slider to 1, and the t slider to 0. Investigate the effect of the parameter t on the point A.

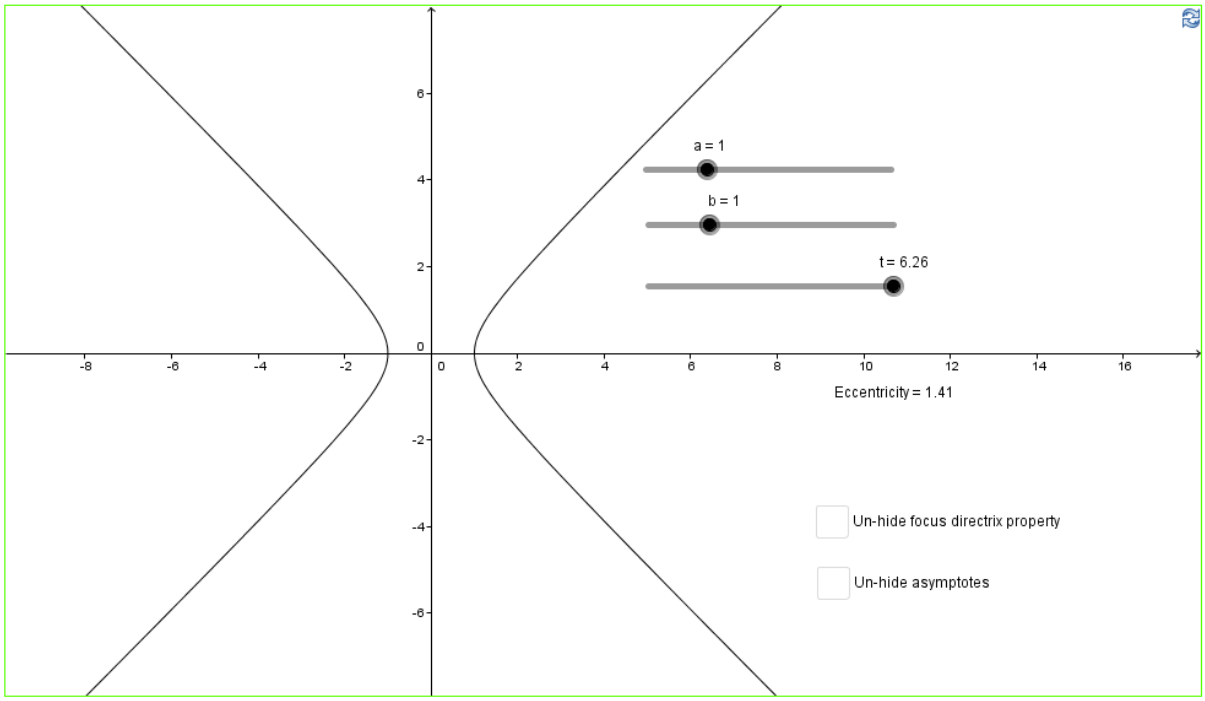
5) Using your findings from question 3 to derive the parametric equations for the curve.

6) Eliminate the parameter t from your parametric equations to give the Cartesian equation of the curve.

7) Click the “Un-hide negative branch” box to display the negative branch of the curve. Describe how this is generated.

Extension – Investigating the Hyperbola

Go to the second applet which presents the hyperbola. It should initially look like this:



The hyperbola shown here is a more general form than the rectangular hyperbola which we study in FP1.

1) The hyperbola is a parametric curve with parameter t. Move the slider labelled t. In what order of the quadrants is the curve plotted?

2) Can you describe the effect of changing a and b?

3) Click the "Un-hide asymptotes" box. Can you work out the equations of the asymptotes in terms of the parameters a and b?

4) The eccentricity, e, of the hyperbola is related to the values of a and b. Describe qualitatively how changing the values of a and b change the value of the eccentricity.

5) Click the "Un-hide focus-directrix property". How does the ratio of the lengths of lines PF1 and BP relate to the eccentricity of the hyperbola?

6) Given that the directrix has equation $x = \frac{a}{e}$ and the point F1 has coordinate $(ae, 0)$ use your answer to question 5 to derive and explicit formula for the eccentricity, e, of a hyperbola.

7) Can you find the parametric equations of the hyperbola? What about the Cartesian equation?