Kirk Hallam Community Academy

A-Level Further Mathematics Year 13
Mechanics Mock 1

AQA Specification

Name: ....

Class: ....

Mark: / 50

1 A point on the edge of a record is rotating at 40 rpm. Given that the diameter of the record is 30 cm, the speed of the point is

12 ms^{-1} 0.63 ms^{-1} 2.63 ms^{-1} 1.26 ms^{-1}

[1 mark]

$40 \text{rpm} = \frac{40 \times 2\pi}{60} = \frac{4\pi}{3} \text{ rad s}^{-1}$

1 | correct answer.

so $\omega = \frac{4\pi}{3} \text{ rad s}^{-1}$

$v = r\omega$

$= 0.3 \times \frac{4\pi}{3}$

$= \frac{2\pi}{5} \approx 1.256637061 \approx 1.26$

2 A particle of mass 2 kg experiences a force of 3 N for 2 seconds. It was initially moving at 2 ms^{-1}. Its speed once the force is removed is

4 ms-1 2 ms^{-1} 3 ms^{-1} 1 ms^{-1}

[1 mark]

1 | correct answer

$I = Ft$

$= 3 \times 6$

$= 6 Ns$

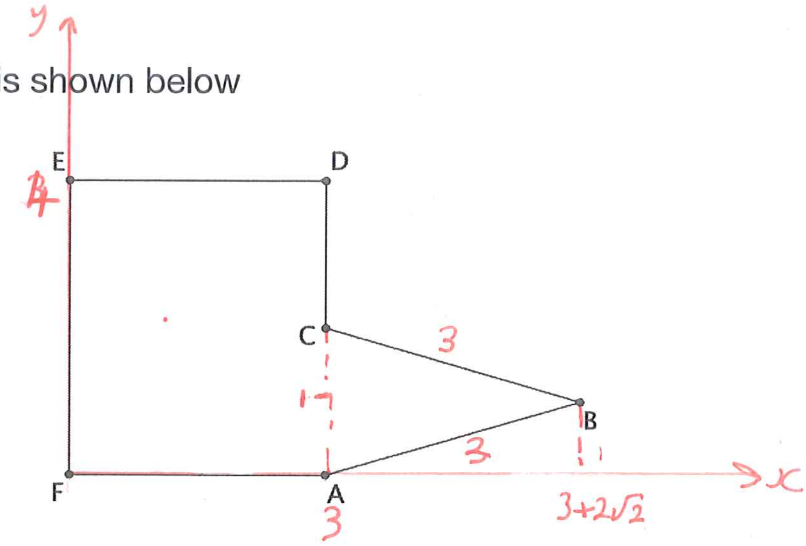
$I = mv - mu$

$\Rightarrow 6 = 3v - 3 \times 2$

$\Rightarrow 3v = 12$

$\Rightarrow v = 4 ms^{-1}$

3 A uniform lamina is shown below



where

$$|AF| = |ED| = 3$$
$$|EF| = 4$$
$$|DC| = 2$$
$$|AB| = |BC| = 3$$

a) Find the distance of the centre of mass of the lamina from

i) EF
ii) AF

[6 marks]

Consider axes as shown ~~belo~~ above

CoM of rectangle $= \left(\frac{3}{2}, 2\right)$ by symmetry

CoM of ~~rectangle~~ triangle:

Lies on $y=1$ by symmetry.

For x coordinate it is $\frac{1}{3}$ of the way from the base so at coordinate $\left(3+\frac{2}{3}\sqrt{2}, 1\right)$

For $\bar{x} = \frac{3+3+3+2\sqrt{2}}{3} = 3+\frac{2}{3}\sqrt{2}$

$\bar{y} = \frac{0+1+2}{3} = 1$

For whole lamina, taking $\rho = 1$

	Area $\times \rho$	$\bar{x}$	$\bar{y}$
▭	12	1.5	2
▷	$2\sqrt{2}$	$3+\frac{2}{3}\sqrt{2}$	1
Total	$12+2\sqrt{2}$	$\bar{x}$	$\bar{y}$

1 | CoM for rectangle
1 | CoM for triangle
1 | Table for whole lamina
1 | Attempt to find $\bar{x}$
1 | Attempt to find $\bar{y}$
1 | Correct centre of mass, given as distances.

So, for $\bar{x}$:

$$(12\times 1.5)+(2\sqrt{2})\times\left(3+\frac{2}{3}\sqrt{2}\right)=(12+2\sqrt{2})\bar{x}$$

$$\Rightarrow \frac{62+18\sqrt{2}}{3}=(12+2\sqrt{2})\bar{x} \quad \Rightarrow \bar{x}\approx 1.97$$

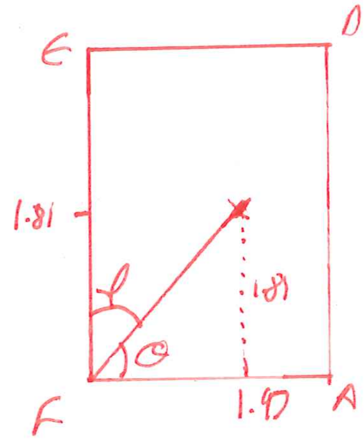
So, for $\bar{y}$: $(12\times 2)+2\sqrt{2}\times 1=(12+2\sqrt{2})\bar{y}$

$$\Rightarrow 24+2\sqrt{2}=(12+2\sqrt{2})\bar{y} \quad \Rightarrow \bar{y}\approx 1.81$$

Hence, CoM of lamina is at $(1.97, 1.81)$

b) The lamina is now suspended from F. Find the angle the side EF makes with the vertical.

[3 marks]



1 | Recognition that line F to CoM is vertical
1 | Attempt to find and angle (that makes sense)
1 | Correct value

So CoM is at a distance of 1.97 from EF and a distance of 1.81 from AF

$$\tan\theta=\left(\frac{1.81}{1.97}\right)$$

$$\Rightarrow \theta = 42.57622866$$ [= 42.6232461 without rounding intermediate values]

So, angle EF makes with the vertical is φ.

$$\varphi = 90-\theta$$
$$= 47.42377134$$
$$\approx 47.4^\circ$$

[= 47.3767538 without rounding intermediate values]

4 An elastic string has natural length 1.5 m and modulus of elasticity 12 N. Calculate the energy stored in the string when it is stretched to a length of 2.3 m.

[3 marks]

$$EPE = \frac{1}{2}\frac{\lambda}{L}x^2$$

$x = 2.3 - 1.5 = 0.8\text{m}$

so $EPE = \frac{1}{2} \times \frac{12}{1.5} \times \left(\frac{4}{5}\right)^2$

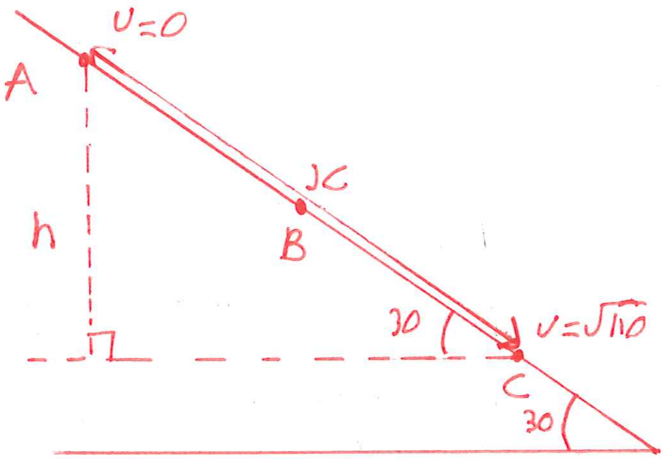
$= \frac{64}{25}$

$= 2.56\text{J}$

5 A particle of mass 3 kg is placed on a smooth plane inclined at $30°$ to the horizontal.
It is released from rest at a point A and moves in a straight line down the plane. It moves past the point B which is 6 m down the plane from A. It subsequently passes the point C which is further down the plane. When at C the speed of the particle is $\sqrt{110}$ ms-1.

Find the distance BC.

[6 marks]



By trigonometry $\sin(30) = \frac{h}{x}$

$\Rightarrow h = x\sin(30)$
$= \frac{x}{2}$

Assuming no air resistance, since the plane is smooth, the gain in kinetic energy is equal to the loss of potential energy.

$$mg\frac{x}{2} = \frac{1}{2}m(\sqrt{110})^2$$

$$\Rightarrow \quad \frac{gx}{2} = 55$$

$$x = \frac{55\times 2}{9.8}$$

$$= \frac{550}{49} \quad [\text{taking } g = 9.8]$$

So the distance BC is

$$|BC| = \frac{550}{49} - 6$$

$$= \frac{256}{49}$$

$$\approx 5.22\,m$$

Suggested guidance:

- Encourage diagrams
- People may use many approaches here.

1	Diagram
1	Correct kinetic energy at C
1	Expression of vertical displacement in terms of distance down the plane
2	Use of conservation of energy
1	Correct answer.

6 Newton's Law of Gravitation states that the attractive force, F, between two point bodies is directly proportional to the product of their masses, m_1 and m_2, and inversely proportional to the square of the distance, r, between them. The constant of proportionality is known as G.

Find the dimensions of G for the formula

$$F = \frac{Gm_1m_2}{r^2}$$

[3 marks]

$$F = \frac{G m_1 m_2}{r^2}$$

$$\Rightarrow \frac{F r^2}{m_1 m_2} = G$$

Hence

$$[G] = \frac{[F][r^2]}{[m_1][m_2]}$$

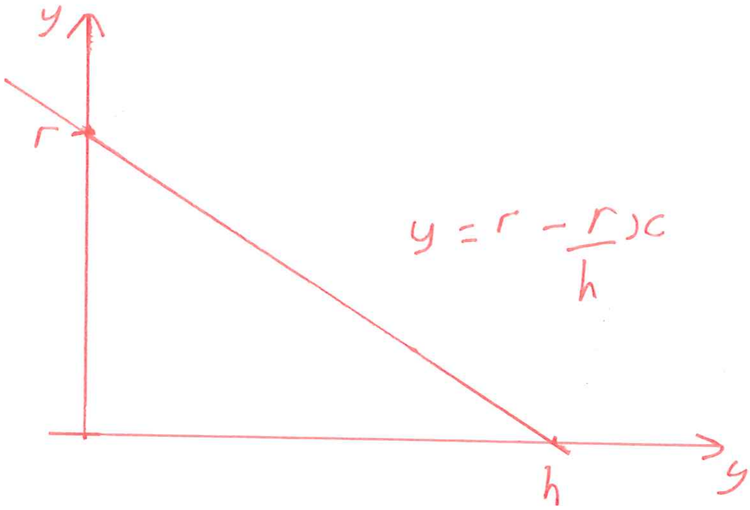
$$= \frac{MLT^{-2}\,L\,L}{M\,M}$$

$$= L^3T^{-2}M^{-1}$$

1 | Rearrangement to $G =$
1 | Correct dimensions for force
1 | Correct dimensions for G

7 a) Use integration to prove that the centre of mass of a solid cone of radius r, and height h lies $\frac{1}{4}$ of the way from the base to the vertex.

[5 marks]



$$\bar{x} = \frac{\pi\int_0^h xy^2\,dx}{\pi\int_0^h y^2\,dx}$$

but $y = \left(r - \frac{r}{h}x\right)$

so $y^2 = \left(r - \frac{r}{h}x\right)^2 = r^2 - \frac{2r^2}{h}x + \frac{r^2}{h^2}x^2$

and $xy^2 = xr^2 - \frac{2r^2x^2}{h} + \frac{r^2}{h^2}x^3$

Hence

$$\bar{x} = \frac{\pi\int_0^h x y^2 \, dx}{\pi\int_0^h y^2 \, dx}$$

$$= \frac{\pi\int_0^h x r^2 - \frac{2r^2}{h}x^2 + \frac{r^2}{h^2}x^3 \, dx}{\pi\int_0^h r^2 - \frac{2r^2}{h}x + \frac{r^2}{h^2}x^2 \, dx}$$

1 | Parametrisation of line to form the cone

1 | Use of CoM $\bar{x} = \frac{\pi\int_0^h x y^2 dx}{\pi\int_0^h y^2 dx}$

$$= \frac{\left[\frac{x^2 r^2}{2} - \frac{2r^2}{3h}x^3 + \frac{r^2}{4h^2}x^4\right]_0^h}{\left[x r^2 - \frac{r^2}{h}x^2 + \frac{r^2}{3h^2}x^3\right]_0^h}$$

1 | Evaluation of integral $\pi\int_0^h x y^2 dx$

$$= \frac{\frac{h^2 r^2}{12}}{\frac{h r^2}{3}}$$

1 | Evaluation of integral $\pi\int_0^h y^2 dx$

$$= \frac{h}{4}$$

1 | $= \frac{h}{4}$

Hence the CoM lies $\frac{1}{4}$ of the way from the base to the vertex.

b) Explain, without calculation why the centre of mass must lie along the perpendicular from the base to the vertex.

[1 mark]

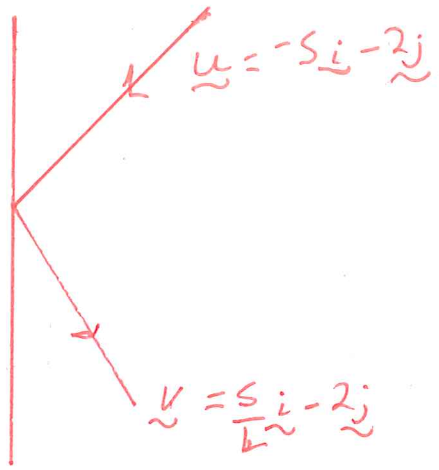
The cone is symmetrical about the x axis.

1 | correct explanation

8 A small smooth ball of mass 1.5 kg is moving in XY plane and collides with a smooth fixed vertical wall containing the y - axis.
The velocity of the ball just before impact is $\mathbf{u} = -5\mathbf{i} - 2\mathbf{j}$ and just after it is $\mathbf{v} = \frac{5}{4}\mathbf{i} - 2\mathbf{j}$.

a) Find the speed of the ball before and after impact.

[2 marks]



$|\underset{\sim}{u}| = \sqrt{(-5)^2 + (-2)^2} = \sqrt{29} \approx 5.39\, ms^{-1}$

$|\underset{\sim}{v}| = \sqrt{\left(\frac{5}{4}\right)^2 + (-2)^2} = \sqrt{\frac{89}{16}} \approx 2.35\, ms^{-1}$

b) Find the loss of kinetic energy as a result of the impact.

[2 marks]

Loss of KE $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$= \frac{1}{2} \times 1.5 \times 29 - \frac{1}{2} \times 1.5 \times \frac{89}{16}$

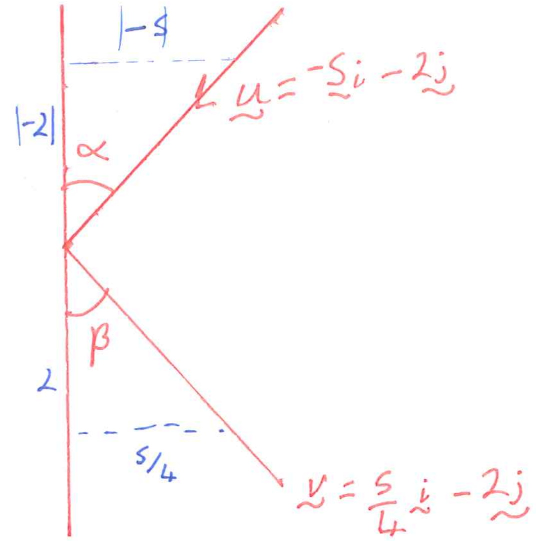
$= \frac{1125}{64}$

$\approx 17.6\,J$

1 | Expression for Loss of KE
1 | $= 17.6\,J$

c) Find the angle of deflection of the ball.

[3 marks]



$\tan(\alpha) = \frac{5}{2}$

$\Rightarrow \alpha = 68.19859051$

$\tan(\beta) = \frac{5/4}{2} = \frac{5}{8}$

$\Rightarrow \beta = 32.00538321$

1 | correct α

1 | correct β

1 | correct angle of deflection.

So

Angle of deflection $= \alpha + \beta$

$= \cancel{32.00}\ 100.2039737$

$\approx 100.2°$

For angle of deflection θ, where

$\cos\theta = \frac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{|\underset{\sim}{u}|\cdot|\underset{\sim}{v}|} = \frac{-9/4}{\sqrt{29}\sqrt{\frac{89}{16}}} \Rightarrow \theta = 100.2°$

d) Show that the coefficient of restitution between the ball and the wall is $\frac{1}{4}$.

[2 marks]

$v_{\perp} = e\, u_{\perp}$

$\Rightarrow \frac{5}{4} = e \cdot 5$

$\Rightarrow e = \frac{1}{4}$

1 | Statement ~~of~~ $v_{\perp} = e\, u_{\perp}$

1 | $e = \frac{1}{4}$

9 Sophie is investigating how the speed, v, of waves on a string depends on the mass, m, and length, l, of the string and the tension, t, in the string.
She conjectures a relationship of the form,

$$v = km^{\alpha}l^{\beta}t^{\gamma},$$

where k is a dimensionless constant.

Determine the values of α, β, and γ.

Excuse incorrect colour here!

[5 marks]

$$[v] = [m^{\alpha}][l^{\beta}][t^{\gamma}]$$

$$\Rightarrow LT^{-1} = M^{\alpha}L^{\beta}(MLT^{-2})^{\gamma}$$

$\underline{L|}$ $\quad 1 = \beta + \gamma \quad$ (1)

$\underline{T|}$ $\quad -1 = -2\gamma \quad$ (2)

$\underline{M|}$ $\quad 0 = \alpha + \gamma \quad$ (3)

(2) $\Rightarrow \gamma = \frac{1}{2}$

$\Rightarrow \alpha = \frac{-1}{2}$

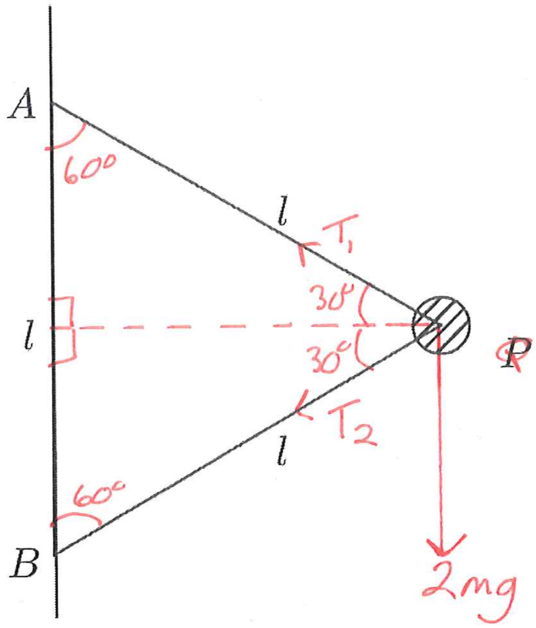
$\Rightarrow \beta = \frac{1}{2}$

Hence,

$$v = km^{-1/2}l^{1/2}t^{1/2}$$

1 | Notice that Tension is a force and dimensionalise accordingly
1 | Compare powers of L, T and M
1 | $\alpha = -1/2$
1 | $\beta = 1/2$
1 | $\gamma = 1/2$

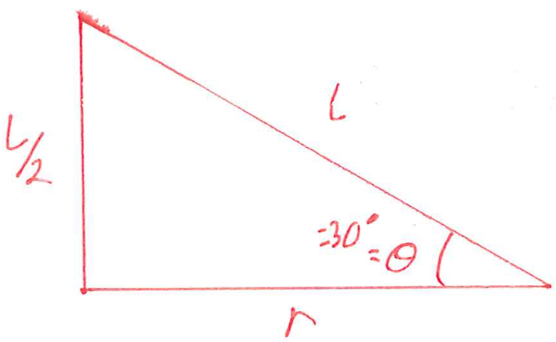
10 A child's clacker consists of a small sphere, Q of mass $2m$ which is joined to two light rods AQ and BQ, each of length l.
The other ends of the rods AQ and BQ are at a distance l apart on a third rod as shown below



AQ and BQ can freely rotate about AB.

Find the tension in both AQ and BQ when the sphere P is moving in a horizontal circle with speed $\sqrt{5gl}$.

[7 marks]



$r = L\cos 30$
$= \frac{\sqrt{3}}{2}L$ (1)

$R\uparrow$

$T_1\cos(60°) = 2mg + T_2\cos(60°)$

$\Rightarrow \quad T_1 - T_2 = 4mg$ (1)

Apply F=ma towards the centre of the circle

$$T_1\cos 30 + T_2\cos 30 = \frac{2mv^2}{r}$$

$$\Rightarrow \frac{\sqrt{3}}{2}T_1 + \frac{\sqrt{3}}{2}T_2 = \frac{2m5gl}{r}$$

$$\Rightarrow \frac{\sqrt{3}}{2}T_1 + \frac{\sqrt{3}}{2}T_2 = \frac{10mgl}{\frac{\sqrt{3}}{2}l} \quad \text{using } (+)$$

$$\Rightarrow \frac{3}{4}T_1 + \frac{3}{4}T_2 = 10mg$$

$$\Rightarrow 3T_1 + 3T_2 = 40mg \quad (2)$$

From (1), $T_1 = 4mg + T_2$, substitute into (2)

$$12mg + 3T_2 + 3T_2 = 40mg$$

$$6T_2 = 28mg$$

$$\Rightarrow T_2 = \frac{14}{3}mg$$

Hence,

$$T_1 = \frac{26}{3}mg$$

1	Finding the radius of motion
1	Attempt to resolve vertically
1	Attempt to apply F=ma towards the centre.
1	Obtaining correct first equation for T_1 and T_2 ($T_1 - T_2 = 4mg$ o.e.)
1	Obtaining 2nd correct equation for T_1 and T_2 ($3T_1 + 3T_2 = 40mg$ o.e.)

1	$T_1 = \frac{26mg}{3}$
1	$T_2 = \frac{14}{3}mg$