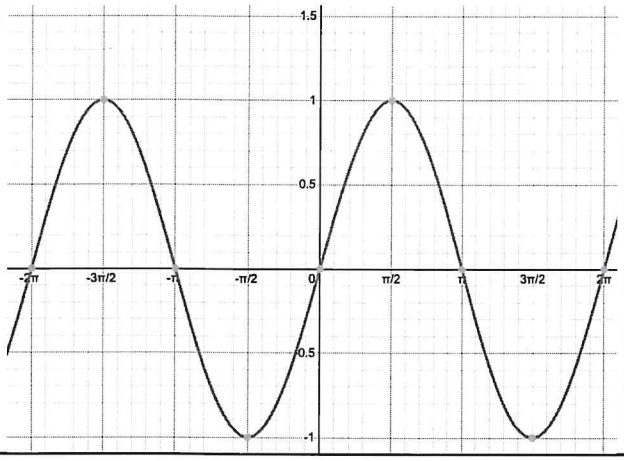
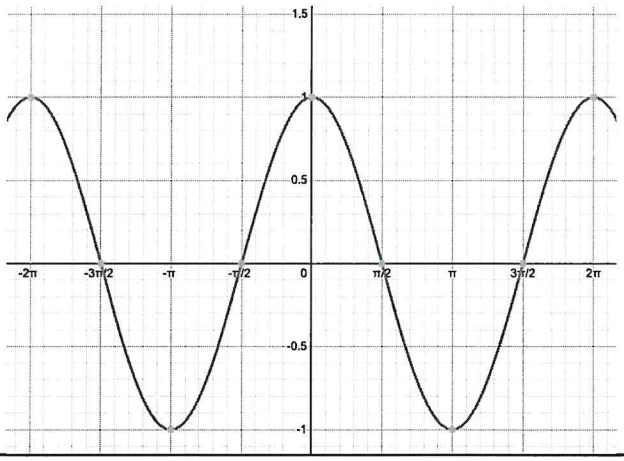
A - Level Maths Coordinate Geometry Recap

Trigonometric Functions

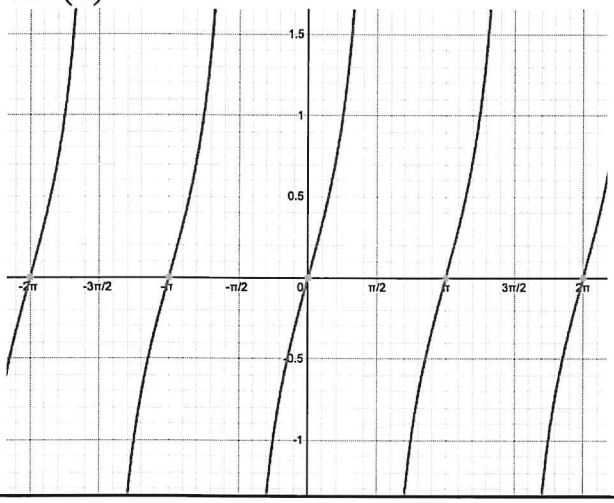
$\sin(x)$



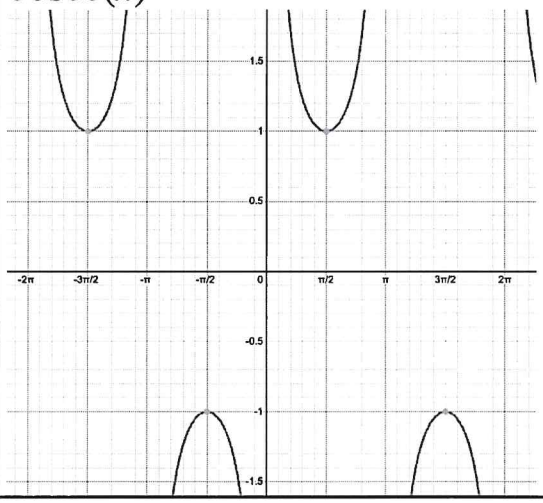
$\cos(x)$



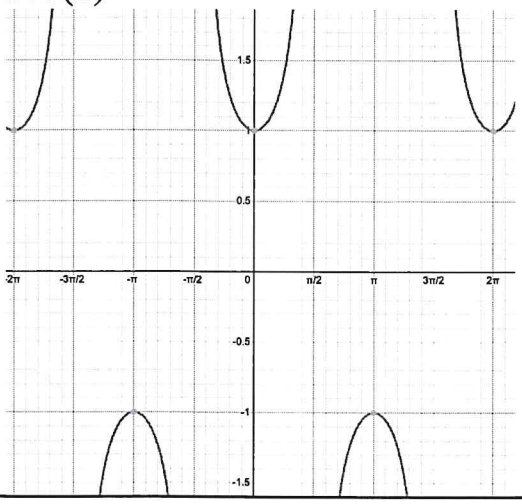
$\tan(x)$



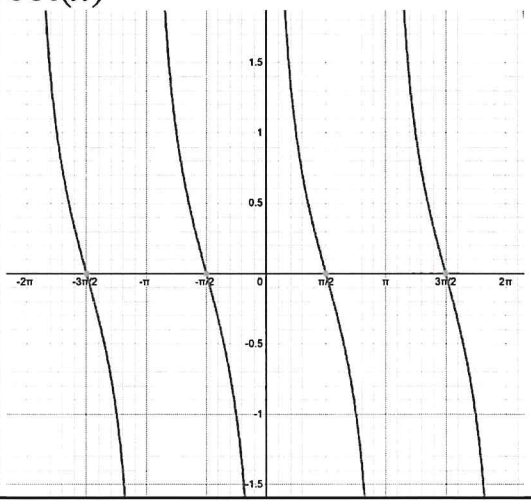
$\text{cosec}(x)$



$\sec(x)$



$\cot(x)$



Trigonometric Identities

$\tan(x) = \dfrac{\sin(x)}{\cos(x)}$

Pythagorean Identities: $\sin^2(x) + \cos^2(x) = 1$

$$\frac{\sin^2(x)}{\sin^2(x)} + \frac{\cos^2(x)}{\sin^2(x)} = \frac{1}{\sin^2(x)}$$

$$1 + \cot^2(x) = \operatorname{cosec}^2(x)$$

$$\frac{\sin^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$\tan^2(x) + 1 = \sec^2(x)$$

Addition Formulae:

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Double Angle Formulae

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

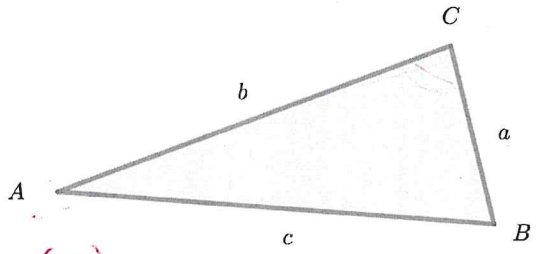
$$= 2\cos^2(x) - 1$$

$$= 1 - 2\sin^2(x)$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

Triangles

For the triangle shown



Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

Sine Rule:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Area:

$$\frac{1}{2}ab\sin(C)$$

Exact Values of Trigonometric Functions:

	0	30 ($\pi/6$)	45 ($\pi/4$)	60 ($\pi/3$)	90 ($\pi/2$)
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$1/\sqrt{2}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

Small angle Approximations:

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$\tan(x) \approx x$$

Example

Express $12\sin(x) - 5\cos(x)$ in the form $R\sin(x-\alpha)$

$$R\sin(x-\alpha) = 12\sin(x) - 5\cos(x)$$

$$R\sin(x)\cos(\alpha) - R\cos(x)\sin(\alpha) = 12\sin(x) - 5\cos(x)$$

$\sin(x)$ | $R\cos(\alpha) = 12$ ①

$\cos(x)$ | $-R\sin(\alpha) = -5 \Rightarrow R\sin(\alpha) = 5$ ②

$$R^2\cos^2(\alpha) + R^2\sin^2(\alpha) = 12^2 + 5^2$$

$$\Rightarrow R^2\underbrace{(\cos^2(\alpha) + \sin^2(\alpha))}_{=1} = 169 \Rightarrow R = 13$$

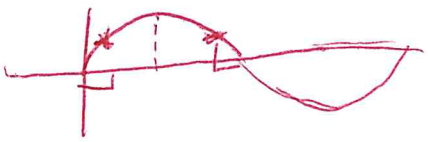
② ÷ ① $\dfrac{R\sin(\alpha)}{R\cos(\alpha)} = \dfrac{5}{12} \Rightarrow \tan(\alpha) = \dfrac{5}{12}$

$$\Rightarrow \alpha = \arctan\left(\frac{5}{12}\right)$$
$$= 0.394791117$$

So $12\sin(x) - 5\cos(x) = 13\sin(x - 0.394)$

Hence, solve $12\sin(x) - 5\cos(x) = 5$

So, $13\sin(x - 0.394) = 5$



$\Rightarrow x - 0.394 = \arcsin\left(\frac{5}{13}\right)$

$= 0.394791117$ and 2.746801534

$\Rightarrow$

$x = 0.789582234, \pi$

Example

Prove $\cot(x) - \tan(x) = 2\cot(2x)$

$$RHS = 2\cot(2x)$$

$$= \frac{2}{\tan(2x)}$$

$$= \frac{2}{\frac{2\tan(x)}{1-\tan^2(x)}}$$

$$= \frac{\cancel{2}(1-\tan^2(x))}{\cancel{2}\tan(x)}$$

$$= \frac{1}{\tan(x)} - \frac{\tan^2(x)}{\tan(x)}$$

$$= \cot(x) - \tan(x)$$

$$= RHS$$

Example

Solve $\sin(\theta) - 2\cos(\theta) = 0$ for $0^\circ \leq \theta \leq 360^\circ$

$$\sin(\theta) = 2\cos(\theta)$$

$$\Rightarrow \frac{\sin(\theta)}{\cos(\theta)} = 2$$

$$\tan(\theta) = 2$$

$$\theta = \arctan(2)$$

$$= 63.43^\circ, \ 243.43^\circ$$

Example

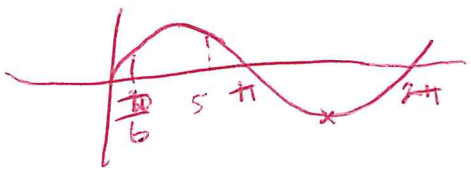
Solve $\sin^2(x) + \sin(x) = \cos^2(x)$ for $0 \leq x \leq 2\pi$

$$\sin^2(x) + \sin(x) = \cos^2(x)$$

$$\sin^2(x) + \sin(x) = 1 - \sin^2(x)$$

$$\Rightarrow 2\sin^2(x) + \sin(x) - 1 = 0$$

$$\Rightarrow (2\sin(x) - 1)(\sin(x) + 1) = 0$$

So $\sin(x) = \frac{1}{2}$ and $\sin(x) = -1$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$