A - Level Maths Coordinate Geometry Recap

Straight Lines / Linear Functions

The equation of a line with gradient m passing through the point (x_1, y_1) has equation

$y = mx + c$, where c is such that $c = y_1 - m_1 x_1$

$$y - y_1 = m(x - x_1)$$

The equation of a line passing through the points with coordinates (x_1, y_1) and (x_2, y_2) has equation

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Consider the line $ax + by + c = 0$

$y = -\frac{a}{b}x - \frac{c}{a}$

$x-$intercept: $x = -\frac{c}{a}$

$y-$intercept: $y = -\frac{c}{b}$

Gradient: $-\frac{a}{b}$

Consider two lines $l_1 : y = m_1x + c_1$ and $l_2 : y = m_2x + c_2$. Then

l_1 and l_2 are parallel if: $m_1 = m_2$

l_1 and l_2 are perpendicular if: $m_1 m_2 = -1$

The midpoint of the line segment joining (x_1, y_1) to (x_2, y_2) is:

$$\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$$

The distance between the point (x_1, y_1) and the point (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example

Find the equation of the perpendicular bisector of the line segment AB where $A(2, -4)$ and $B(6,4)$

Midpoint of $AB = \left(\frac{2+6}{2} , \frac{-4+4}{2} \right) = (4, 0)$

Gradient of AB is $\frac{\Delta y}{\Delta x} = \frac{4 - -4}{6-2} = \frac{8}{4} = 2$

$\therefore$ Gradient of perpendicular bisector is $-\frac{1}{2}$

so eq'n of $\perp$ bisector is of the form

$$y = -\frac{1}{2}x + c$$

Passing through $(4, 0)$

$$0 = -\frac{1}{2} \times 4 + c$$

$$\Rightarrow c = 2$$

So Eq'n of bisector is

$$y = -\frac{1}{2}x + 2$$

Circles

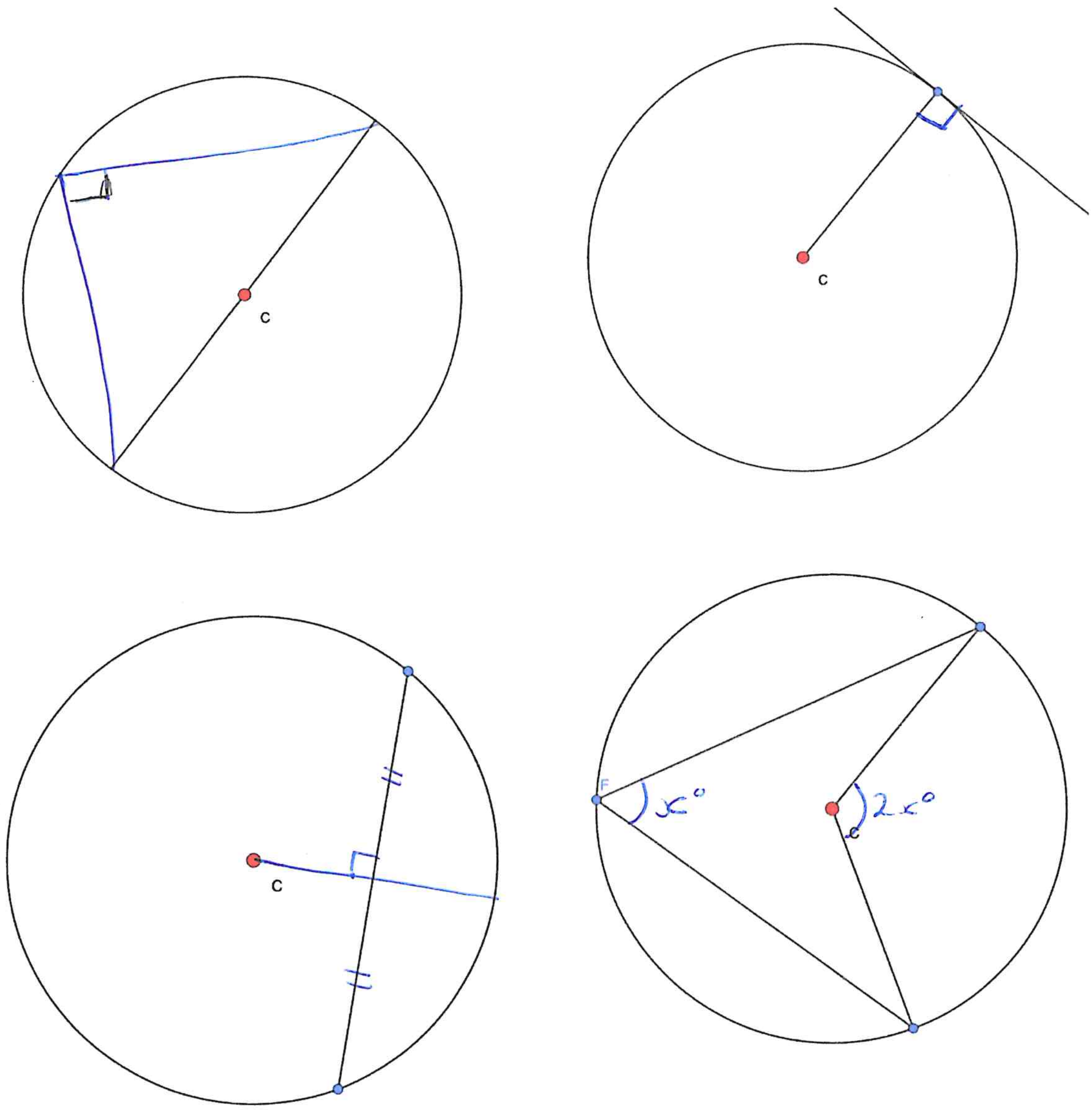
The circle, centre the origin radius r has equation:

$$x^2 + y^2 = r^2$$

The circle, centre (a, b), radius r has equation:

$$(x-a)^2 + (y-b)^2 = r^2$$

Annotate the circles below to complete the circle theorems



The tangent at (h, k) to the circle with equation $x^2 + y^2 = a^2$ has equation:

$hx + ky = a^2$

Example

Find the centre and radius of the circle $x^2 - 12x + y^2 + 6y - 4 = 0$

$x^2 - 12x + y^2 + 6y - 4 = 0$

$\Rightarrow (x-6)^2 + (y+3)^2 - 36 - 9 - 4 = 0$

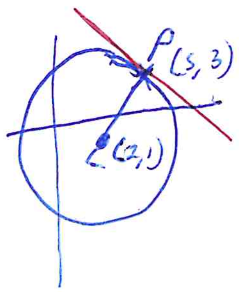
$\Rightarrow (x-6)^2 + (y+3)^2 = 49$

Centre : $(6, -3)$

Radius : 7

Example

Find the equation of the tangent to the circle $(x-2)^2 + (y+1)^2 = 25$ at the point $(5,3)$.



Centre : $(2, -1)$

Gradient CP : $\frac{3--1}{5-2} = \frac{4}{3}$

so gradient of tangent is $-\frac{3}{4}$

$y = -\frac{3}{4}x + c$

Passing through $(5,3)$

$3 = -\frac{3}{4} \times 5 + c \Rightarrow c = 3 + \frac{15}{4}$

$= \frac{27}{4}$

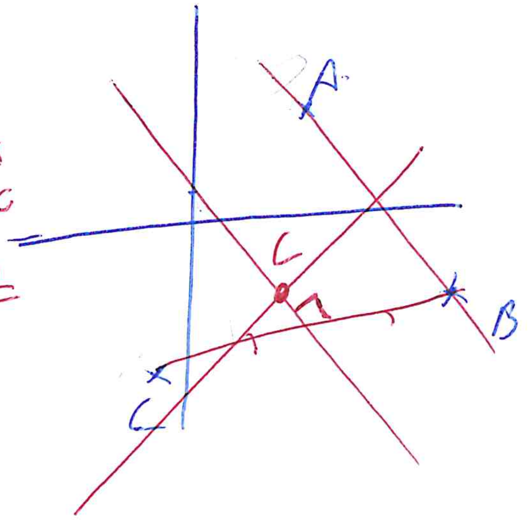
So equation of tangent is $y = -\frac{3}{4}x + \frac{27}{4}$

Example

A circle passes through the points $A(8,8)$, $B(16,-4)$ and $C(-2,-16)$. Find the equation of the circle given that the equation of the perpendicular bisector of BC is $3x+2y=1$.

$$y = -\frac{3}{2}x + \frac{1}{2}$$

Gradient of $AB = \dfrac{-4-8}{16-8} = \dfrac{-12}{8}$



So gradient of the bisector is $\dfrac{2}{3}$

Midpoint of $(AB) = \left(\dfrac{16+8}{2}, \dfrac{8+{-4}}{2}\right) = (12, 2)$

So eq^n of bisector of AB is $y = \dfrac{2}{3}x + c$, passing through $(12, 2)$

$$2 = \frac{2}{3}\times 12 + c \Rightarrow c = -6$$

Hence eq^n of bisector of AB is

$$y = \frac{2}{3}x - 6$$

$$\Rightarrow 3y = 2x - 18 \quad \Rightarrow -2x + 3y = -18$$

Solve
$$-2x + 3y = -18 \quad ①$$
$$3x + 2y = 1 \quad ②$$

$$\Rightarrow x = 3,\ y = -4$$

So the centre, C is $(3, -4)$

To find the radius, consider

$$|CA| = \sqrt{5^2 + 12^2} = 13$$ so radius = 13

So

$$(x-3)^2 + (y+4)^2 = 169$$