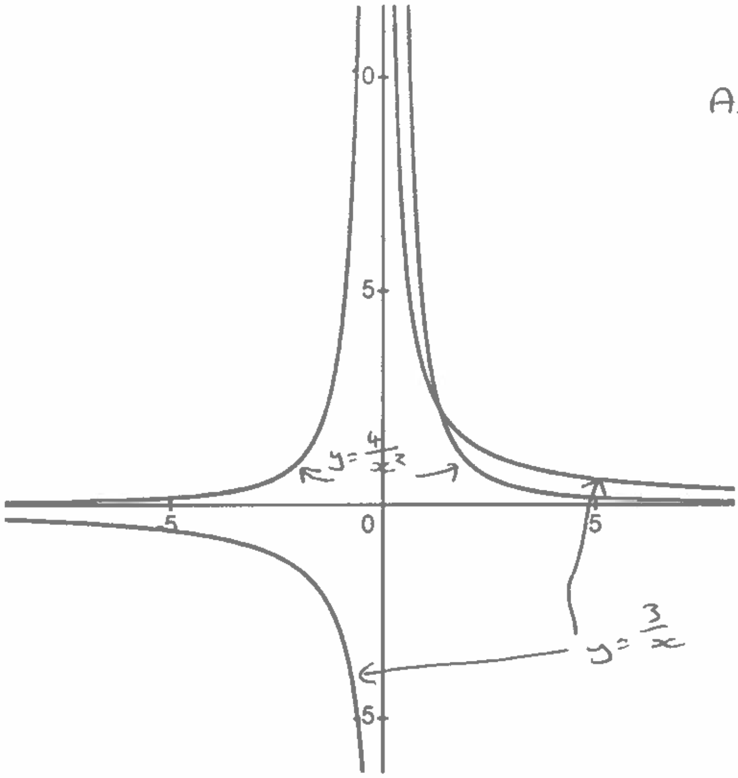
(1)



Asymptotes $y = 0$

$x = 0$

(2) $\underline{u} = \vec{OA} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$ $\quad \underline{v} = \vec{OB} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$ $\quad \underline{w} = \vec{AB}$

$= \vec{OB} - \vec{OA}$

$= \underline{v} - \underline{u}$

$= \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$\vec{AM} = \frac{1}{2}\underline{w}$

$= \frac{1}{2}(\underline{v} - \underline{u})$

$= \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$|AM| = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$

(3) $\dfrac{(2x+3)^{1/2}}{3+4x} = (2x+3)^{1/2}(3+4x)^{-1}$

$(2x+3)^{1/2} = 3^{1/2}\left(1 + \frac{2}{3}x\right)^{1/2} \approx \sqrt{3}\left(1 + \left(\frac{1}{2}\right)\left(\frac{2}{3}x\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{1 \times 2}\left(\frac{2}{3}x\right)^2\right)$

$= \sqrt{3}\left(1 + \frac{1}{3}x - \frac{4}{72}x^2\right)$

$= \sqrt{3} + \frac{\sqrt{3}}{3}x - \frac{\sqrt{3}}{18}x^2$

Valid for $-3/2 < x < 3/2$

③ continued...

$$(3+4x)^{-1} = 3^{-1}\left(1+\frac{4}{3}x\right)^{-1}$$

$$\approx \frac{1}{3}\left(1 + (-1)\left(\frac{4}{3}x\right) + \frac{(-1)(-2)}{1\times 2}\left(\frac{4}{3}x\right)^2\right)$$

$$= \frac{1}{3}\left(1 - \frac{4}{3}x + \frac{16}{9}x^2\right)$$

$$= \frac{1}{3} - \frac{4}{9}x + \frac{16}{27}x^2$$

Valid for $-\frac{3}{4} < x < \frac{3}{4}$

$$\frac{(2x+3)^{1/2}}{(3+4x)} = (2x+3)^{1/2}(3+4x)^{-1}$$

$$= \left(\sqrt{3} + \frac{\sqrt{3}}{3}x - \frac{\sqrt{3}}{18}x^2 + \dots\right)\left(\frac{1}{3} - \frac{4}{9}x + \frac{16}{27}x^2 + \dots\right)$$

$$= \frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{3}x + \frac{23\sqrt{3}}{54}x^2$$

Valid for $-\frac{3}{4} < x < \frac{3}{4}$

④ $$y = \frac{3x^2+7x+3}{e^x}$$

$f(x) = 3x^2+7x+3$
$f'(x) = 6x+7$
$g(x) = e^x$
$g'(x) = e^x$

$$\frac{dy}{dx} = \frac{e^x(6x+7) - e^x(3x^2+7x+3)}{(e^x)^2}$$

$$= \frac{e^x(6x+7-3x^2-7x-3)}{e^{2x}}$$

$$= \frac{-3x^2-x+4}{e^x}$$

(4) Continued...

At a stationary point, $\frac{dy}{dx} = 0$

$$\frac{-3x^2 - x + 4}{e^x} = 0 \quad \Rightarrow \quad -3x^2 - x + 4 = 0$$

$$x = 1, \ -\frac{4}{3}$$

$x = 1, \quad y = \frac{13}{e}$ $\qquad \left(1, \frac{13}{e}\right)$

$x = -\frac{4}{3}, \quad y = \frac{-1}{e^{-4/3}} = -e^{4/3}$ $\qquad \left(-\frac{4}{3}, -e^{4/3}\right)$

(5) $p(x) = x^4 + ax^3 + bx^2 - 15x + 18$

$p(2) = 0 \quad \Rightarrow \quad 2^4 + a(2)^3 + b(2)^2 - 15(2) + 18 = 0$

$$8a + 4b + 4 = 0$$

$p(-3) = 0 \quad \Rightarrow \quad (-3)^4 + a(-3)^3 + b(-3)^2 - 15(-3) + 18 = 0$

$$-27a + 9b + 144 = 0$$

$$\Rightarrow \quad a = 3, \quad b = -7$$

(6) $\sec^2 x + \operatorname{cosec}^2 x$

$$= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$$

$$= \frac{1}{\cos^2 x \sin^2 x}$$

$$= \frac{1}{\cos^2 x} \times \frac{1}{\sin^2 x} = \sec^2 x \operatorname{cosec}^2 x$$ as required

(7) $f(x) = x^3 + 3x^2$

$f(x+h) = (x+h)^3 + 3(x+h)^2$

$= x^3 + 3x^2h + 3xh^2 + h^3 + 3x^2 + 6xh + 3h^2$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 3x^2 + 6xh + 3h^2 - x^3 - 3x^2}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 + 6xh + 3h^2}{h}$$

$$= \lim_{h\to 0} 3x^2 + 3xh + h^2 + 6x + 3h$$

$$= 3x^2 + 6x$$

(8) l_1: $y = 2x + c$ $(5, 2)$

$2 = 2(5) + c$

$\Rightarrow c = -8$ $\boxed{y = 2x - 8}$

c_1: $(x-4)^2 + (y-2)^2 = 5^2$

sub in $y = 2x - 8$

$(x-4)^2 + (2x-10)^2 = 25$

$x^2 - 8x + 16 + 4x^2 - 40x + 100 = 25$

$5x^2 - 48x + 91 = 0$

$x = 7, \frac{13}{5}$

Points $(7, 6)$ and $\left(\frac{13}{5}, \frac{-14}{5}\right)$

(9)

$a + d = 9$

$a + 7d = 51$

$6d = 42 \implies d = 7$

$a = 2$

$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$

$S_{20} = \frac{1}{2}(20)\left[2(2) + (20-1) \times 7\right]$

$= 1370$

(10)

$y = \sin x$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

Point $\left(\frac{\pi}{6}, \frac{1}{2}\right)$

$\frac{dy}{dx} = \cos x$

$\left.\frac{dy}{dx}\right|_{x=\pi/6} = \cos \pi/6 = \frac{\sqrt{3}}{2} = m$

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$ ← equation of tangent

x axis, $y = 0$.

$-\frac{1}{2} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$

$-1 = \sqrt{3}\left(x - \frac{\pi}{6}\right)$

$-\frac{1}{\sqrt{3}} + \frac{\pi}{6} = x$

$\left(-\frac{\sqrt{3}}{3} + \frac{\pi}{6}, 0\right)$

$= \left(-\frac{2\sqrt{3}+\pi}{6}, 0\right)$

y axis, $x = 0$.

$y - \frac{1}{2} = \frac{\sqrt{3}}{2}\left(-\frac{\pi}{6}\right)$

$y = -\frac{\sqrt{3}\pi}{12} + \frac{1}{2}$

$= \frac{-\sqrt{3}\pi + 6}{12}$

$\left(0, \frac{-\sqrt{3}\pi + 6}{12}\right)$

(11) $x = \cos u$

$\frac{dx}{du} = -\sin u \implies dx = -\sin u \, du$

$$\int -\frac{1}{\sqrt{1-x^2}} \, dx$$

$$= \int -\frac{1}{\sqrt{1-\cos^2 u}} \times -\sin u \, du$$

$$= \int \frac{\sin u}{\sqrt{\sin^2 u}} \, du$$

$$= \int \frac{\sin u}{\sin u} \, du$$

$$= \int 1 \, du$$

$x = \cos u$

$\implies u = \arccos x$

$$= u + c$$

$$= \arccos x + c$$

(12) $x = n^3 - n$

If n is an odd integer, then $n = 2m+1$

for some integer m.

$$x = (2m+1)^3 - (2m+1)$$

$$= 8m^3 + 12m^2 + 4m$$

$$= 4(2m^3 + 3m^2 + m)$$

$$= 4m(2m^2 + 3m + 1)$$

$$= 4m(m+1)(2m+1)$$

This is a multiple of 4, so x will be a multiple of 12 if $m(m+1)(2m+1)$ is a multiple of 3

(12) continued...

$k \in \mathbb{Z}$

There are 3 possibilities for m:

* m is a multiple of 3, ie $m = 3k$
* m is one more than a multiple of 3, ie $m = 3k+1$
* m is two more than a multiple of 3, ie $m = 3k+2$

If $m = 3k$, $x = 4(3k)(3k+1)(6k+1)$

$= 12(k)(3k+1)(6k+1)$

$\therefore$ multiple of 12

If $m = 3k+1$, $x = 4(3k+1)(3k+2)(2(3k+1)+1)$

$= 4(3k+1)(3k+2)(6k+3)$ multiple of 3

$= 12(3k+1)(3k+2)(2k+1)$

$\therefore$ multiple of 12

If $m = 3k+2$, $x = 4(3k+2)(3k+3)(2(3k+2)+1)$

$= 12(3k+2)(k+1)(6k+5)$

$\therefore$ multiple of 12

As this exhausts all possibilities ~~for~~ for m, x will be a multiple of 12 for all odd values of $n > 2$