AQA A-Level Further Mathematics Warmup - Paper 3MD 2023

Find the work done when the force $F(t) = t^2 + 5t$ acts for 3 seconds.

An elastic string has modulus of elasticity λ and natural length l. Prove that the work done extending from extension x_1 to x_2 is $\frac{\lambda}{2l}\left(x_2^2 - x_1^2\right)$

A printer can make cards and gift tags. The number, x, of gift tags must be no more than 80 and the number of cards, y, must be no more than 60. A gift tag costs £1 to produce and a card £3. This week the printer can spend no more than £120.

a) Formulate the situation described on the left as a linear programming problem.

b) Represent this graphically.

c) Suppose that the printer makes 20p profit on gift tag and 75p profit on a card. Find the maximum profit.

A particle A of mass 2 kg is attached to the lower end of a light inextensible string with the upper end fixed at a point B. When the particle moves in a horizontal circular path, the string traces out the curved surface of a cone and makes an angle $60°$ with the downward vertical. The centre of the circular path lies 2 m directly below B

a) Find the tension in the string

b) Find the angular speed of the particle

In critical path analysis what is a "critical activity" ?

What does Newton's Experimental law state in the context of collisions?

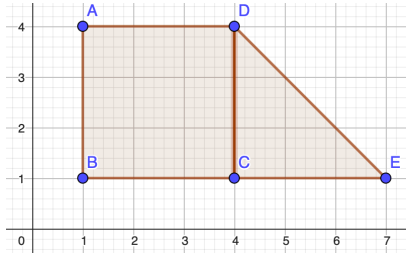
Construct the Cayley table for the set $\{1,3,5,7\}$ under multiplication modulo 8

Find the centre of mass of the lamina shown below.
If it is suspended from A, find the angle the vertical makes with the side AD

What does it mean for a binary operation $\star$ on a set S to be associative and commutative?

A uniform rod AB of mass 2000 grams is pivoted at A and held in equilibrium at an angle of $45°$ to the vertical by a force F applied at B, perpendicular to AB. Find the force F

G is a connected planar graph with 6 faces and 4 vertices. How many edges will G have?



What is the route inspection problem?

What does Kuratowski's theorem say?

The pay-off matrix from Player 1's point of view is shown to the right. What would be the pay-off matrix from Player 2's point of view:

	P2 plays A	P2 plays B
P1 plays A	2	0
P1 plays B	8	-4

Draw the graph K_5

Is the equation $v^2 = u^2 + 2gs$ dimensionally consistent ?

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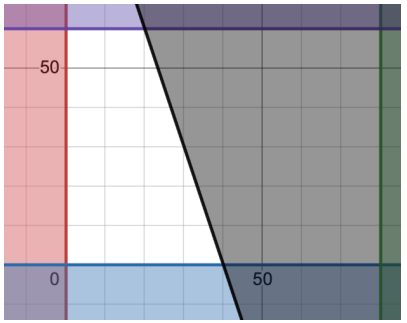
$$\text{Impulse} = \int_0^3 t^2 + 5t \ \mathrm{d}t$$
$$= \left[\frac{t^3}{3} + \frac{5}{2}t^2\right]_0^3$$
$$= 31.5 \text{ N}$$

$$\text{WD} = \int_{x_1}^{x_2} T \, \mathrm{d}x$$
$$= \int_{x_1}^{x_2} \frac{\lambda x}{l}$$
$$= \frac{\lambda}{l}\left[\frac{x^2}{2}\right]_{x_1}^{x_2}$$
$$= \frac{\lambda}{2l}\left(x_2^2 - x_1^2\right)$$

$$x \geq 0$$
$$y \geq 0$$
$$x \leq 80$$
$$y \leq 60$$
$$3x + y \leq 120$$

Optimal solution = $(20,60)$, so 20 gift tags and 60 cards.



a) Resolving vertically,
$$T\cos(6) - 2g = 0$$
$$\Rightarrow \quad T = \frac{2g}{0.5}$$
$$= 39.2 \text{ N}$$

An activity where there is no slack.

$$e = \frac{\text{speed of separation}}{\text{speed of approach}}$$

$\times_8$	1	3	5	7
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

$$\frac{27}{2}\bar{x} = 9 \times \frac{5}{2} + \frac{9}{2} \times 5$$
$$\text{so } \bar{x} = \frac{10}{3}$$

$$\frac{27}{2}\bar{y} = 9 \times \frac{5}{2} + \frac{9}{2} \times 2$$

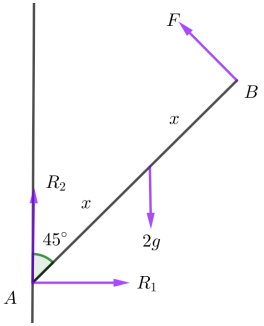
$$\text{So } \bar{y} = \frac{7}{3}.$$

The angle the side AD makes with the vertical is
$$\theta = \arctan\left(\frac{5}{7}\right) \approx 35.54°$$

b) Applying $F = ma$ towards the centre.
$$T\sin(60) = 2a \Rightarrow a = \frac{T\sin(60)}{2}$$
$$\text{so } a = \frac{49\sqrt{3}}{5}.$$
Now $\omega^2 = \sqrt{\frac{a}{r}}$, so
$$\omega = \sqrt{\frac{16.97}{1.5\tan 60}} \approx 2.55$$

Commutative:
$a \star b = b \star a$

Associative:
$(a \star b) \star c = a \star (b \star c)$



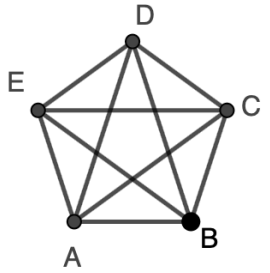
By Euler's formula
$F + V - E = 2$ and so
G has 8 edges.

To find the shortest route that covers all of the arcs of a graph at least once, returning to the starting point if required.

A graph is non-planar if and only if it contains a subgraph that is a subdivision of either $K_{3,3}$ or K_5

Taking anticlockwise moments about A.
$$2gx\cos(45) - F \times 2x = 0$$
$$F = \frac{2g\cos 45)}{2}$$
So,
$$= \frac{49\sqrt{2}}{10}$$
$$\approx 6.92 \text{ N}$$

	P1 plays A	P1 plays B
P2 plays A	2	0
P2 plays B	0	-4



$$[v^2] = L^2T^{-2}$$
$$[u^2] = L^2T^{-2}$$
$$[2gs] = LT^{-2}L = L^2T^{-2}$$

And so the equation
$v^2 = u^2 + 2gs$ is dimensionally consistent.