

AQA Level 2 Further Maths 2023 Paper 1

Do not turn over the page until instructed to do so.

This assessment is out of 80 marks and you will be given 90 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: / 80



1 Expand and fully simplify

$$(3x + 4)(2x - 1)(x + 3)$$

[3 marks]

$$\begin{aligned}
 & (3x + 4)(2x - 1)(x + 3) \\
 &= (3x + 4)(2x^2 - x + 6x - 3) \\
 &= (3x + 4)(2x^2 + 5x - 3) \\
 &= 6x^3 + 15x^2 - 9x + 8x^2 + 20x - 12 \\
 &= 6x^3 + 23x^2 + 11x - 12
 \end{aligned}$$

2 Circle the value of $\cos^2(30^\circ)$.

$$\cos(30) = \frac{\sqrt{3}}{2}$$

[1 mark]

$\frac{3}{4}$

$\frac{1}{4}$

3

$\frac{1}{2}$

3 Rationalise and simplify fully $\frac{\sqrt{5}}{7 - \sqrt{5}}$

[3 marks]

$$\begin{aligned}\frac{\sqrt{5}}{7 - \sqrt{5}} &= \frac{\sqrt{5}}{7 - \sqrt{5}} \times \frac{(7 + \sqrt{5})}{(7 + \sqrt{5})} \\ &= \frac{7\sqrt{5} + 5}{49 - 7\sqrt{5} + 7\sqrt{5} - 5} \\ &= \frac{7\sqrt{5} + 5}{44} \\ &= \frac{1}{44} (5 + 7\sqrt{5})\end{aligned}$$

4 Find $\frac{dy}{dx}$ for $y = 3x^2 \left(x^2 + 2x + \frac{1}{x} \right)$

[4 marks]

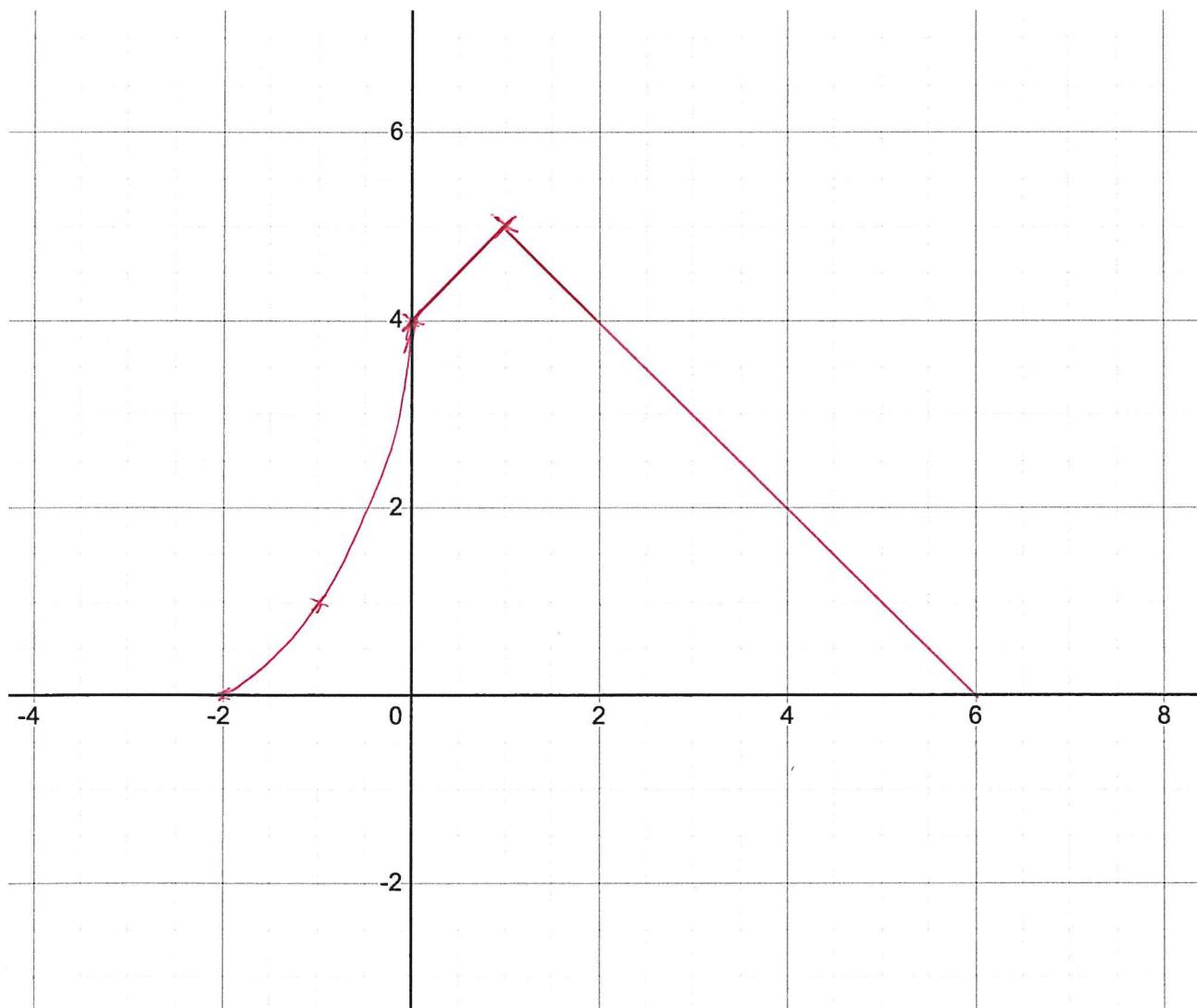
$$\begin{aligned}y &= 3x^2 \left(x^2 + 2x + \frac{1}{x} \right) \\ &= 3x^4 + 6x^3 + 3x^2 \\ \text{So } \frac{dy}{dx} &\leq 12x^3 + 18x^2 + 3\end{aligned}$$

- 5 Let $f(x)$ be defined as follows.

$$\begin{aligned}f(x) &= x^2 + 4x + 4 & -2 \leq x < 0 \\&= 4 + x & 0 \leq x < 1 \\&= 6 - x & 1 \leq x < 6\end{aligned}$$

On the grid, draw $y = f(x)$

[4 marks]



- 5 Circle centre $(-3, 2)$ passes through the point $P(0, 6)$.

a) Find the equation of the circle.

[2 marks]

$$(x+3)^2 + (y-2)^2 = r^2$$

$$\text{but } r = \sqrt{3^2 + 4^2} = 5$$

$$\text{so } (x+3)^2 + (y-2)^2 = 25$$

b) Find the equation of the tangent to the circle at P .

[4 marks]

Let C be the centre. Then

$$\text{gradient } CP = \frac{4}{3}$$

So gradient of tangent is $-\frac{3}{4}$ and it passes through $(0, 6)$.

Hence

$$6 = -\frac{3}{4} \times 0 + C \Rightarrow C = 6$$

$$\text{so } y = -\frac{3}{4}x + 6$$

$$\Rightarrow 3x + 4y = 24$$

is an equation of the tangent

- c) The tangent crosses the x -axis at the point Q . Find the area of the triangle OQP , where O denotes the origin.

[4 marks]

Tangent crosses x -axis when $y=0$, i.e.

$$3x + 6 = 0 \\ x = -2$$

Hence, area of $OQP = \frac{1}{2} \times 8 \times 6$

$$= \frac{1}{2} \times 48$$

$$= 24 \text{ square units}$$

- 6 Work out the distance between the points $A(-4, 6)$ and $B(2, 3)$.

[2 marks]

$$|AB| = \sqrt{(2 - (-4))^2 + (3 - 6)^2}$$

$$= \sqrt{6^2 + (-3)^2}$$

$$= \sqrt{36 + 9}$$

$$= \sqrt{45} < 3\sqrt{5}$$

- 7 The first four terms of a quadratic sequence are

1 10 23 40

Work out an expression for the n th term of the sequence.

[3 marks]

$$\begin{array}{cccc} 1 & 10 & 23 & 40 \\ \underbrace{+9}_{4} & \underbrace{+13}_{4} & \underbrace{+17}_{4} & \end{array}$$

So quadratic sequence with a $2n^2$

n	1	2	3	4
seq	1	10	23	40
$2n^2$	2	8	18	32
$\text{seq} - 2n^2$	-1	2	5	8

Consider

$$\begin{array}{cccc} -1 & 2 & 5 & 8 \\ \underbrace{+3}_{+3} & \underbrace{+3}_{+3} & \underbrace{+3}_{+3} & \end{array}$$

$$3n - 4$$

Hence, the n th term of the original quadratic sequence is

$$2n^2 + 3n - 4$$

8 Expand $(2 + 3x)^4$.

[4 marks]

$$\begin{aligned}
 (2+3x)^4 &= 1 \times 2^4 \times (3x)^0 + 4 \times 2^3 \times (3x)^1 \\
 &\quad + 6 \times (2^2) \times (3x)^2 + 4 \times (2^1) \times (3x)^3 \\
 &\quad + 1 \times (3x)^4 \\
 &= 16 + 96x + 216x^2 + 216x^3 + 81x^4
 \end{aligned}$$

9 Under the transformation represented by the matrix $\begin{pmatrix} 4 & -1 \\ 6 & -1 \end{pmatrix}$ which of the following points is not invariant?

[1 mark]

(1,3)

(2,4)

(3,9)

(5,13)

$$\begin{pmatrix} 4 & -1 \\ 6 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -1 \\ 6 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -1 \\ 6 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -1 \\ 6 & -1 \end{pmatrix} \begin{pmatrix} 5 \\ 13 \end{pmatrix} = \begin{pmatrix} 7 \\ 17 \end{pmatrix}$$

should be (5,11)
which is
invariant.
Corrected on
uploaded paper

- 10 Work out the value of b such that $(x^4)^b = (x^3)^5$,

[2 marks]

$$x^{4b} = x^5$$

$$4b = 15$$

$$\Rightarrow b = \frac{15}{4}$$

- 11 Simplify fully $\frac{3x^2 - 27}{x + 2} \div \frac{x^2 - 6x + 9}{x^2 + 5x + 6}$

[4 marks]

$$\frac{3(x^2 - 9)}{x+2} \div \frac{(x-3)(x-3)}{(x+2)(x+3)}$$

$$= \frac{3(x-3)(x+3)}{x+2} \times \frac{(x+2)(x+3)}{(x-3)(x-3)}$$

$$= \frac{3(x+3)^2}{x-1}$$

- 12 Let $f(x) = 4x + 1$ and $g(x) = x^2$. Solve $fg(x) = gf(x)$.

[4 marks]

$$\begin{aligned} fg(x) &= f(g(x)) = f(x^2) \\ &= 4x^2 + 1 \end{aligned}$$

$$\begin{aligned} gf(x) &= g(f(x)) = g(4x+1) \\ &= (4x+1)^2 \\ &= 16x^2 + 8x + 1 \end{aligned}$$

Hence, $fg(x) = gf(x)$

$$\Rightarrow 4x^2 + 1 = 16x^2 + 8x + 1$$

$$\begin{aligned} \Rightarrow 0 &= 12x^2 + 8x \\ &= 4x(3x+2) \end{aligned}$$

Hence, $x = 0$ or $\frac{-2}{3}$

- 13 The n th term of a sequence is given by $\frac{4n^2}{3n^2 + 3}$.

a) One term of the sequence is $\frac{64}{51}$. Work out the value of n .

[2 marks]

$$\frac{4n^2}{3n^2+3} = \frac{64}{51}$$

$$\Rightarrow 204n^2 = 192n^2 + 192$$

$$\Rightarrow 12n^2 = 192$$

$$n^2 = 16$$

$$\therefore \text{So, } n = 4$$

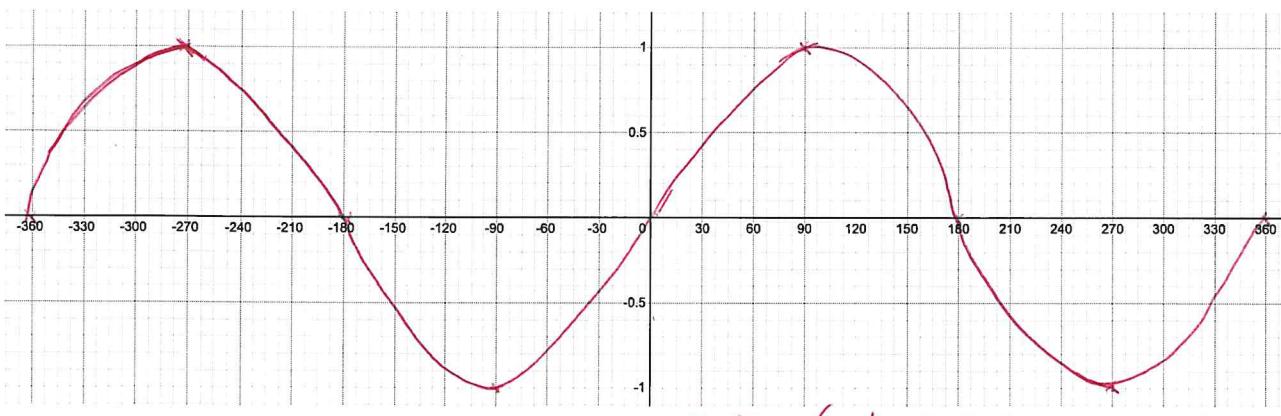
- b) Write down the limiting value of the sequence as $n \rightarrow \infty$.

[1 mark]

$$\frac{4}{3}$$

- 14 a) On the axes below sketch $y = \sin(x)$ for $-360^\circ \leq x \leq 360^\circ$

[2 marks]



b) Solve $\sin(x) = \frac{\sqrt{3}}{2}$ for $-360^\circ \leq x \leq 360^\circ$

[4 marks]

$$\sin(x) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = \arcsin\left(\frac{\sqrt{3}}{2}\right) \\ = 60^\circ$$

\therefore in the range $-360^\circ \leq x \leq 360^\circ$

$$x = -240^\circ, -300^\circ, 60^\circ, 120^\circ$$

15 Prove that

$$\frac{1}{1 + \sin(x)} + \frac{1}{1 - \sin(x)} = \frac{2}{\cos^2(x)}$$

[4 marks]

Consider the LHS

$$\begin{aligned} \frac{1}{1 + \sin(x)} + \frac{1}{1 - \sin(x)} &= \frac{(1 - \sin(x)) + (1 + \sin(x))}{(1 + \sin(x))(1 - \sin(x))} \\ &= \frac{2}{\cos^2(x)} \end{aligned}$$

16 Let n be a positive integer.

- a) Write down the next odd number greater than $2n - 1$.

[1 mark]

$$2n + 1$$

- b) Prove that the product of two consecutive odd numbers is always one less than a multiple of 4.

[3 marks]

$$\begin{aligned}(2n-1)(2n+1) &= 4n^2 - 2n + 2n - 1 \\&= 4n^2 - 1\end{aligned}$$

which is one less than a multiple of 4

$$17 \quad \begin{pmatrix} 4 & a \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 22 \\ b \end{pmatrix}$$

Find the values of a and b .

[3 marks]

$$\begin{aligned} 12 + 2a &= 22 \quad (1) \\ 6 + 2 &= b \quad (2) \end{aligned}$$

$$(2) \Rightarrow b = 8$$

$$\begin{aligned} (1) \Rightarrow 2a &= 10 \\ \Rightarrow a &= 5 \end{aligned}$$

$$\text{So, } a = 5, b = 8$$

- 18 a) Using the factor theorem show that $(x - 2)$ is a factor of
 $p(x) = x^3 + x^2 - 4x - 4.$

[2 marks]

$$\begin{aligned} p(2) &= 8 + 4 - 8 - 4 \\ &= 0 \end{aligned}$$

Since $p(2) = 0$, by the factor theorem $(x - 2)$
is a factor of $p(x)$

- b) Hence, fully factorise $p(x)$.

[3 marks]

$$\begin{aligned} x^3 + x^2 - 4x - 4 &= (x - 2)(x^2 + 3x + 2) \\ &= (x - 2)(x + 2)(x + 1) \end{aligned}$$

19 Solve the simultaneous equations

$$\begin{aligned}x + y &= \frac{23}{5} & (1) \\xy &= -2 & (2)\end{aligned}$$

[5 marks]

Let $y = -\frac{2}{x}$ from (2) substitute into (1)

$$x - \frac{2}{x} = \frac{23}{5}$$

$$\Rightarrow x^2 - 2 = \frac{23}{5}x$$

$$\Rightarrow 5x^2 - 10 = 23x$$

$$5x^2 - 10 = 23x$$

$$\Rightarrow 5x^2 - 23x - 10 = 0$$

$$\Rightarrow (5x+2)(x-5) = 0$$

$$\text{so } x = -\frac{2}{5}, \text{ or } x = 5$$

$$\text{When } x = -\frac{2}{5}, y = 5$$

$$\text{When } x = 5, y = -\frac{2}{5}$$

- 20 A triangle has sides 6cm, 8cm and 10cm.

Given that the vertices of the triangle all lie on the circumference go a circle, find the exact area of the circle not covered by the triangle.

[6 marks]

$$\text{Noting that } 8^2 + 6^2 = 64 + 36 \\ = 100 \\ = 10^2$$

by the converse of Pythagoras theorem, the triangle ABC is right angled

So, by circle theory AB must be a diameter
 \Rightarrow radius = 5.

Then

$$\text{Arcade} = 25\pi$$

$$\begin{aligned}\text{Area}_{\text{triangle}} &= \frac{1}{2} \times 6 \times 8 \\ &= 24\end{aligned}$$

Hence, the area not covered is $25\pi - 24$