

AQA A-Level Maths 2023 Paper 3

Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

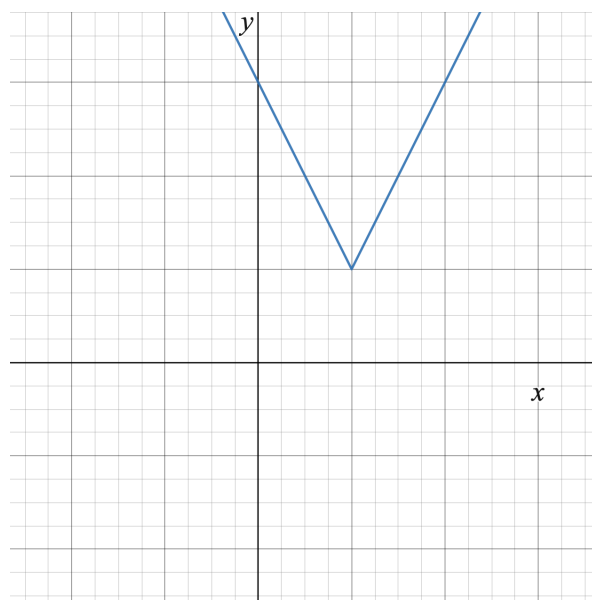
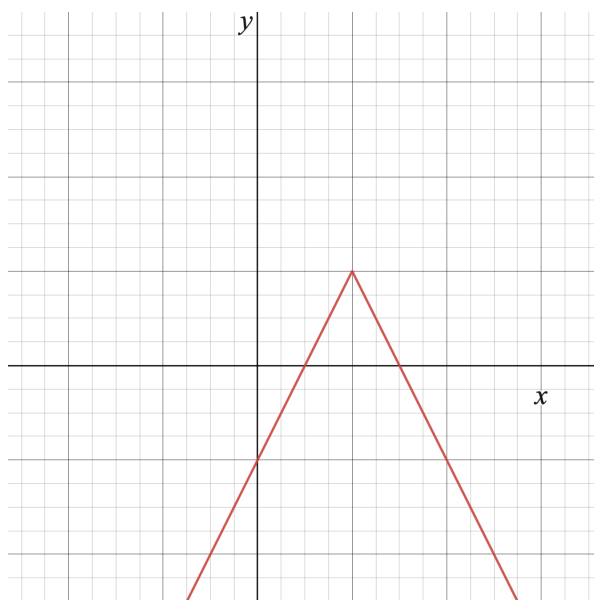
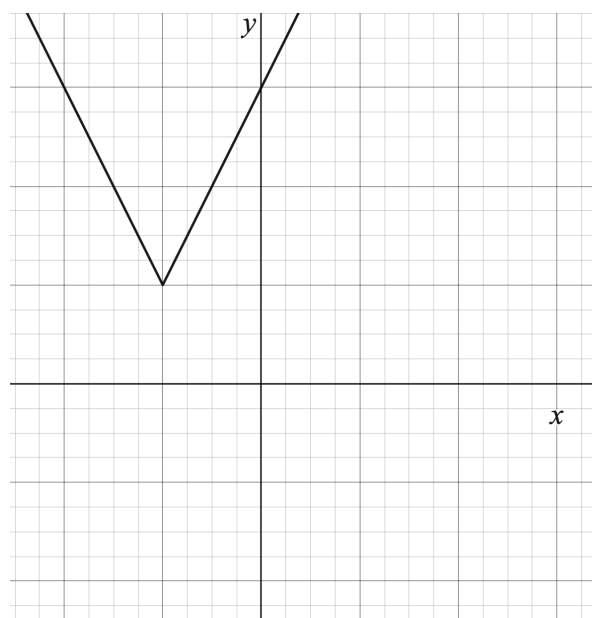
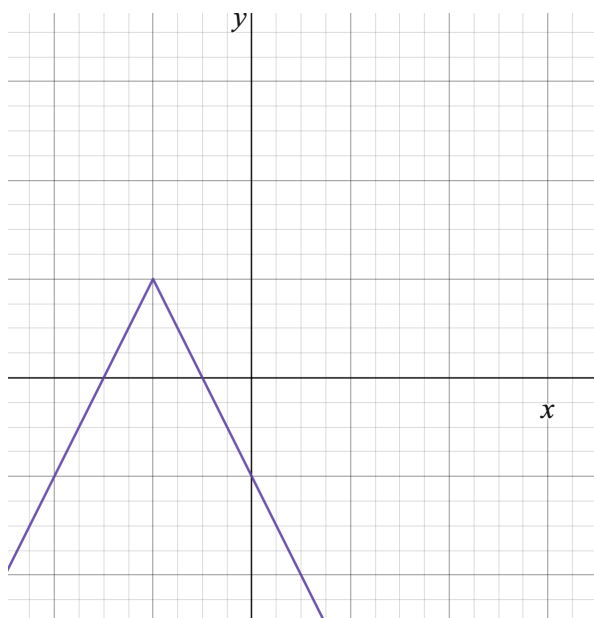
Name:

Total Marks: **/ 100**



SECTION A

- 1 Which of the graphs below is a graph of $y = 2 - |2x + 4|$?



[1 mark]

- 2 How many times do the graphs of $y = \cot(x)$ and $y = \sec(2x)$ intersect in the interval $0 \leq x \leq 2\pi$?

0

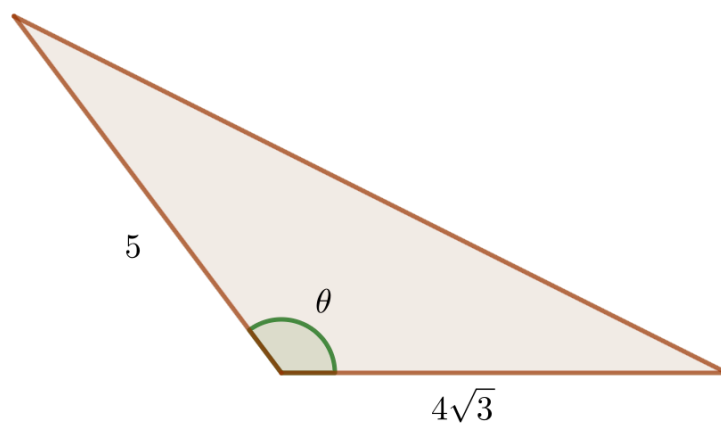
1

2

3

[1 mark]

- 3 The area of the triangle below is 15 square units.



Find the value of θ .

$$\frac{2\pi}{3}$$

$$\frac{\pi}{3}$$

$$\frac{\pi}{6}$$

$$\frac{5\pi}{6}$$

[1 mark]

4 Show that $\sum_{r=1}^5 \ln \left(\frac{r}{r+2} \right) = -\ln(21)$

[3 marks]

5 Differentiate, from first principles $f(x) = x^3 + 2x$

[5 marks]

- 6** The rate of change of the radius of a circle is inversely proportional to the radius cubed.

a) Show that the rate at which the area of the circle, A , changes

satisfies $\frac{dA}{dt} = \frac{2\pi^2}{A}$

[4 marks]

b) Explain why $\frac{dA}{dt} > 0$

[1 mark]

7 Consider the cubic function $p(x) = x^3 + x^2 + kx + 3k$.

a) Given that $x = 3$ is a root of $p(x)$, find k

[2 marks]

b) Prove that $p(x)$ has only one real root.

[4 marks]

- 8** Find the solution of the equation $\cos(\theta) + 3 \cos\left(\theta + \frac{\pi}{6}\right) = 0$ in the range $0 \leq \theta \leq 2\pi$

[6 marks]

9

a) If $x = 4 \sin(u)$, simplify $\sqrt{16 - x^2}$

[2 marks]

b) Hence, or otherwise, find $\int_1^3 \frac{1}{x^2 \sqrt{16 - x^2}} dx$.

[8 marks]

10 Consider the circle $x^2 - 8x + y^2 - 8y + 7 = 0$

a) What is the centre, C , of the circle?

[1 mark]

b) Find the equation of the tangent to the circle at $P(8,7)$

[3 marks]

- c) Another tangent to the circle at $Q(7,0)$ has equation $3x - 4y = 21$.

This tangent meets the tangent found in (a) at the point R .

Shape T is formed by removing the sector CPQ of the circle from the quadrilateral $CPRQ$.

Find the ratio

Perimeter of T : Area of T

[8 marks]

SECTION B

- 10** The table below shows the probability distribution of a discrete random variable X .

x	1	2	3	4
$P(X = x)$	$2k$	0.4	0.3	$5k$

Find the value of k

$$\frac{3}{70}$$

$$\frac{3}{10}$$

$$1$$

$$\frac{3}{20}$$

[1 mark]

- 11** Let X be a continuous random variable such that $X \sim N(23, 0.28)$.

Then the standard deviation of X is, correct to 2 decimal places,

$$0.28$$

$$0.53$$

$$0.08$$

$$6.44$$

[1 mark]

12 A histogram of the following data has been drawn

Range	Frequency
$0 \leq x < 20$	32
$20 \leq x < 30$	20
$30 \leq x < 35$	x
$35 \leq x < 50$	y

The height of the first bar, when plotted, is 8 cm and there are 124 data points in total.

- a)** The bar representing the range 30 – 35 is 12 cm high. Find x .

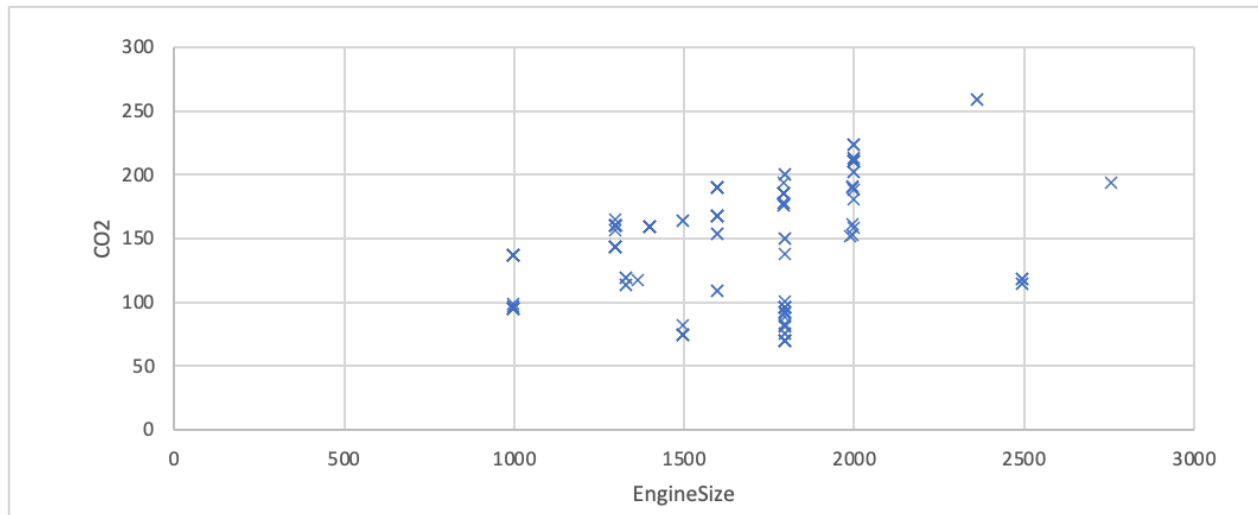
[2 marks]

- b)** Find the frequency density for the range 35 to 50.

[2 marks]

- 13** Miles is using the large data set to investigate whether there is a correlation between engine size and CO_2 emissions.

Looking at a single manufacturer he produces the scatter plot below.



- a)** What kind of correlation is shown in the scatter graph above.

[1 marks]

- b)** Miles concludes, using data from the large data set, that for all cars in the UK, engine size can be used to predict CO_2 emissions from cars.

Explain why, using your knowledge of the large data set why this may be an unsafe conclusion to make.

[2 marks]

15 A factory in Germany produces metal ferrules for pencils.

They should have a mean diameter of 7.4 mm.

A sample of 10 ferrules are taken and found to have a sample mean diameter of 6.6 mm.

Is there evidence, at the 5% significance level to suggest that the mean diameter of the ferrules is less than it should be.

You may assume that the population variance is 1 mm².

[5 marks]

- 16** Liala is driving to school in the morning and has to pass through 5 sets of traffic lights on her journey.

Overtime she has estimated that the probability of the lights being against her (i.e. red) is 0.6.

- a)** Find the probability of zero sets of the traffic lights being red.

[1 mark]

- b)** Find the probability of there being more than 3 sets of traffic lights on red.

[2 marks]

She estimates that each red light adds 70 seconds to her journey.

- c)** Find the expected extra journey time due to waiting at lights.

[2 marks]

17 A random variable X is normally distributed.

If $P(X \leq 175) = 0.7222$ and $P(X > 190) = 0.1729$, find the mean μ and variance σ^2 , of X

[6 marks]

- 18** Amir is seeking the views of the Sixth Form on the introduction of a uniform.

There are 260 students in the Sixth Form in total with 140 in Year 12.

He decided to use a random stratified sample to obtain his data.

- a)** State one advantage and one disadvantage of stratified sampling.

[2 marks]

Advantage:

Disadvantage:

- b)** Amir wants a sample of size 65. Determine how many Year 12s and Year 13s Amir should sample.

[3 marks]

- c)** Explain how Amir could collect the random sample of Year 12 students.

[3 marks]

- d)** In practice stratified sampling can be hard to carry out for large populations. What kind of sampling could be used instead?

[1 mark]

19 A and B are two events such that

$$P(A \cap B) = 0.15$$

$$P(A' \cap B') = 0.4$$

$$P(A) = 2P(B)$$

a) Find $P(A)$

[4 marks]

b) Find $P(A | B)$

[2 marks]

c) Are A and B independent?

[1 mark]

- 20 a)** Where does the maximum of the normal distribution $X \sim N(120, 3^2)$ occur?

[1 mark]

- b)** Where do the inflection points of X occur?

[1 mark]

- b)** Let $Y \sim N(\mu, \sigma^2)$. Then Y has probability distribution function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

By differentiating find the location of the inflection points of the distribution.

[6 marks]

