## AQA A-Level Maths 2023 Paper 2

Do not turn over the page until instructed to do so.
This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your full name below

## Name:

## Total Marks: / 100

## SECTION A

1 Which of the graphs below shows the graph of $y=\sec \left(x-\frac{\pi}{2}\right)$ ?





2 What is the domain of $f(x)=\frac{3}{\sqrt{2+x}}$ ?

$$
\begin{array}{ll}
x \in \mathbb{R} & x \in \mathbb{R}: x>-2 \\
x \in \mathbb{R}: x \neq-2 & x \in \mathbb{R}: x>2
\end{array}
$$

[1 mark]

3 The derivative of $-\cos ^{2}(x)$ with respect to $x$ is

$$
\begin{array}{ll}
\sin (2 x) & -\sin (2 x) \\
-2 \cos (x) & -2 \cos (x) \sin (x)
\end{array}
$$

4 Prove that $2^{2 n}-1$ is divisible by 3
[4 marks]

5 Differentiate, from first principles $f(x)=\cos (2 x)$
[5 marks]

6 Consider the function defined parametrically by the equations

$$
x(t)=2+2 \cos (t), \quad y(t)=3+\sin (t)
$$

By finding the cartesian equation of the graph of this function determine whether it intersects the $x$ - axis or the $y$ - axis.

7 Find $\int \frac{\sqrt{x}}{1+\sqrt{x}} \mathrm{~d} x$
[8 marks]

8 Consider the polynomial $p(x)=4 x^{4}+a x^{3}+b x^{2}-23 x-6$.
a) Given that $(x+3)$ and $(x-2)$ are factors of $p(x)$, find, showing all reasoning, the values of $a$ and $b$.
[4 marks]
b) Hence, fully factorise $p(x)$.
[2 marks]
c) Using your results from (a) and (b), find $\int \frac{x+3}{p(x)} \mathrm{d} x$.
[6 marks]

9 Consider the function $f(x)=\frac{x}{\sqrt{3 x-3}}$
a) Using the quotient rule, show that $f^{\prime}(x)=\frac{x-2}{2 \sqrt{3}(x-1)^{\frac{3}{2}}}$
[4 marks]
b) Show that the graph of $y=f(x)$ has exactly one point of inflection.
[7 marks]
c) For what values of $x$ is $f(x)$ concave?
[1 mark]

## SECTION B

10 A number of forces act on a particle so that the resultant force is
$\binom{2}{4}$
$N$. One force acting on the particle is $\binom{5}{1} \mathrm{~N}$.

The total of the other forces acting on the particle is

$$
\begin{array}{ll}
\binom{-3}{3} & \binom{7}{5} \\
\binom{3}{-3} & \binom{-7}{-5}
\end{array}
$$

[1 mark]

11 A box sits of a rough horizontal surface. The coefficient between the box and the floor is 0.8 .

The box weighs 375 N
Given that the box is on the point of moving, find the force required to overcome the friction.
468.75 N

600 N
300 N
93.75 N

12 Using the velocity time graph shown below derive the SUVAT equation $v^{2}=u^{2}+2 a s$.

[4 marks]

13 In this question use $g=10 \mathrm{~ms}^{-2}$.
A particle is projected from the point with coordinates $(1,1)$ with an initial velocity of $\binom{5}{4} \mathrm{~ms}^{-1}$ and moves freely under gravity.
a) Show that the position vector, $\mathbf{r}(t)$, of the particle $t$ seconds after it has been projected is given by

$$
\mathbf{r}(t)=\binom{1+5 t}{1+4 t-5 t^{2}}
$$

[2 marks]
b) Find the position vector of the particle when it reaches its maximum height.
[3 marks]
c) Hence, find the distance between the particle's starting position and its maximum height.

14 A particle moves in a straight line with velocity, as a function of $t$,

$$
v(t)=2 t^{2}+t \cos (t)+3
$$

a) Find an expression for the acceleration of the particle at time $t$.

## [2 marks]

b) Show, that for the acceleration found in (a), when $t=\frac{5 \pi}{6}$ the acceleration is $\frac{35 \pi}{12}-\frac{\sqrt{3}}{2}$
c) Find an expression for the displacement, from the origin, of the particle at time $t$. You may use that when $t=0, x(0)=2$.
[4 marks]

15 Two cameras are positioned on the M1 northbound so that they are 3 km apart.

A car passes the first camera at $35 \mathrm{~ms}^{-1}$. The car continues at this speed for 25 seconds, and then decelerates uniformly until it passes the second camera at $28 \mathrm{~ms}^{-1}$.
a) Calculate the distance travelled by the car in the first 25 seconds.
[2 marks]
b) Find the time for which the car is decelerating.
[3 marks]
c) How long does it take, in total, for the car to travel this 3 km stretch of the motorway?
d) Sketch a velocity-time graph for this motion.
e) The speed limit on the motorway is 70 mph . The driver wants to know if he will be caught for speeding.

Determine whether it makes a difference if the cameras were average speed cameras between the two locations or traditional "instant" speed cameras.
[4 marks]

16 A block of 5 kg is placed on a rough plane inclined at $\alpha^{\circ}$ to the horizontal where $\cos (\alpha)=\frac{3}{5}$.

The coefficient of friction between the block and the plane is $\frac{2}{5}$.
One end of a light rope is attached to the block and passes over a smooth pulley at the top of the plane. The other end of the rope is attached to a sphere of mass 6 kg which hangs vertically below the pulley.

The system is released from rest, and as the sphere falls the block moves directly up the plane with acceleration $a \mathrm{~ms}^{-2}$.
a) What is the implication from the pulley being smooth?
[1 mark]
b) Draw a labelled force diagram for this situation
c) Find an equation of motion for the sphere.
[1 mark]
d) Determine the acceleration of the block in terms of $g$.
[6 marks]
e) After two seconds the sphere hits the floor and the string becomes slack.

Find the minimum distance between the blocks starting position and the pulley for the block to not hit the pulley.
[4 marks]

