

AQA A-Level Maths 2023 Paper 1B

Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

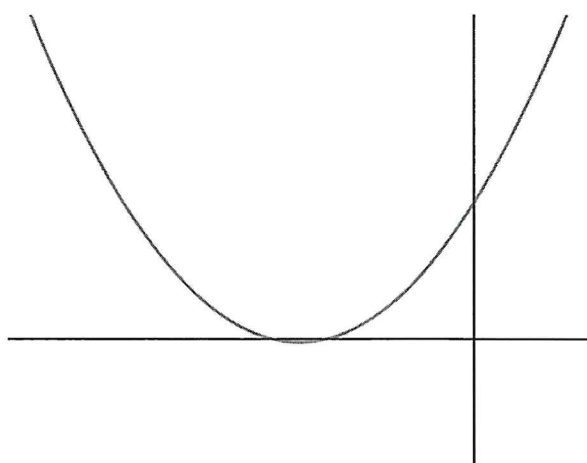
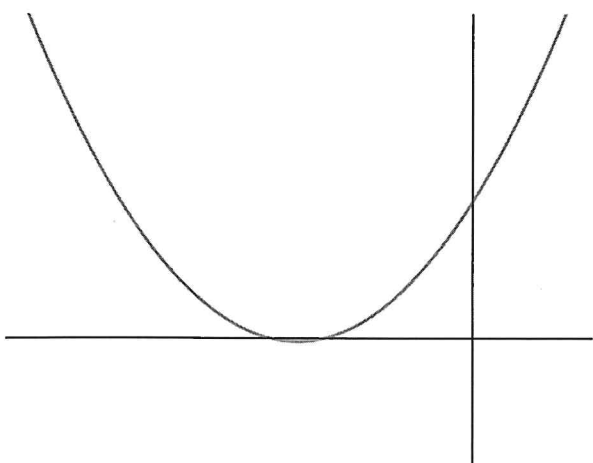
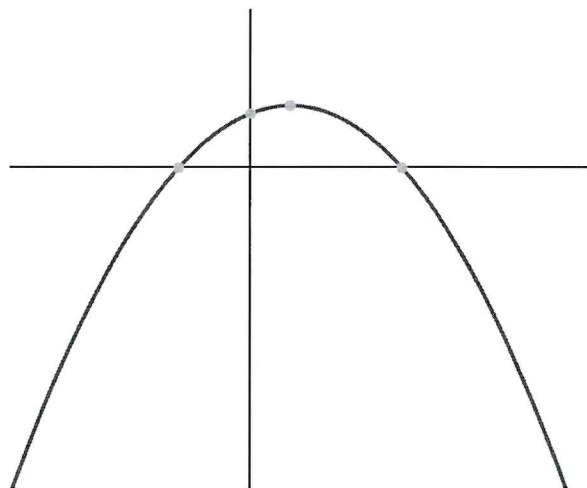
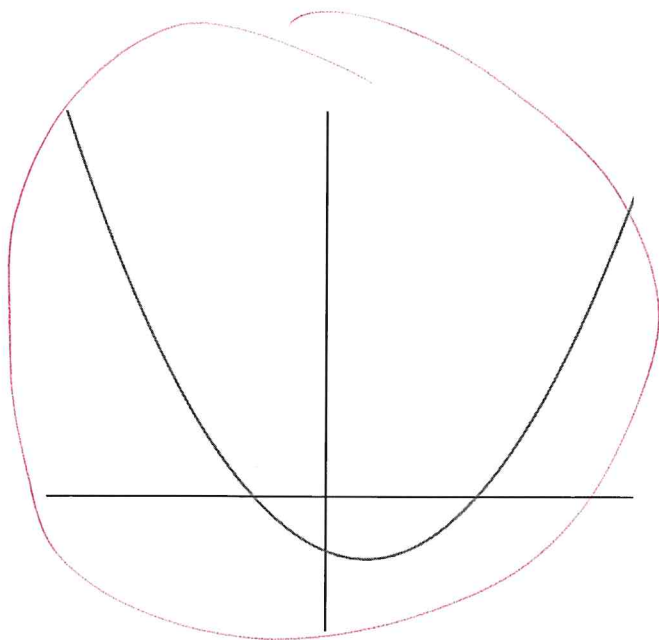
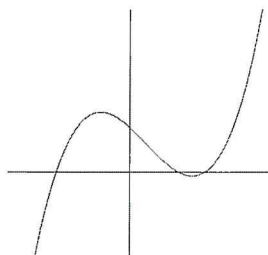
Name:

Total Marks: **/ 100**

Anyues



- 1 Which is a possible gradient function of the gradient function shown.



[1 mark]

2 The second derivative of $y = \ln(x)$ is

$$\frac{1}{x}$$

$$-\frac{1}{x}$$

$$-\frac{1}{x^2}$$

$$\frac{1}{x^2}$$

[1 mark]

3 Find the binomial coefficient of x^2 in the expansion of $(5 - 2x)^5$

$$5000$$

$$-2000$$

$$-5000$$

$$2000$$

[1 mark]

- 4 Triangle ABC has area $\frac{63\sqrt{3}}{4}$. Given that $|AB| = 7\sqrt{3}$ and $\angle ABC = 60^\circ$ find $|BC|$.

[4 marks]

$$|BC| = \frac{9}{\sqrt{3}}$$

- 5 The quadratic below has two real roots.

$$(2k - 1)x^2 + (7k + 1)x + 9k = 0$$

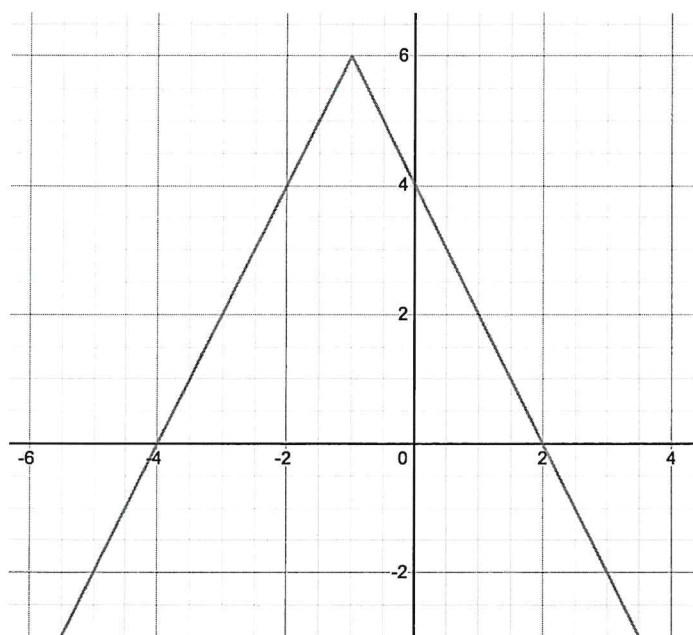
Find the range of possible values of k .

[8 marks]

$$\frac{25 - 18\sqrt{2}}{23} \leq k \leq \frac{25 + 18\sqrt{2}}{23}$$

6 a) Sketch $y = 6 - |2x + 2|$

[2 marks]



b) Solve the inequality $6 - |2x + 2| > 2$

[3 marks]

$$-3 < x < 7$$

- 7 a) Find, and classify, the stationary points of
 $y = x^3 + 3x^2 - 24x + 18$

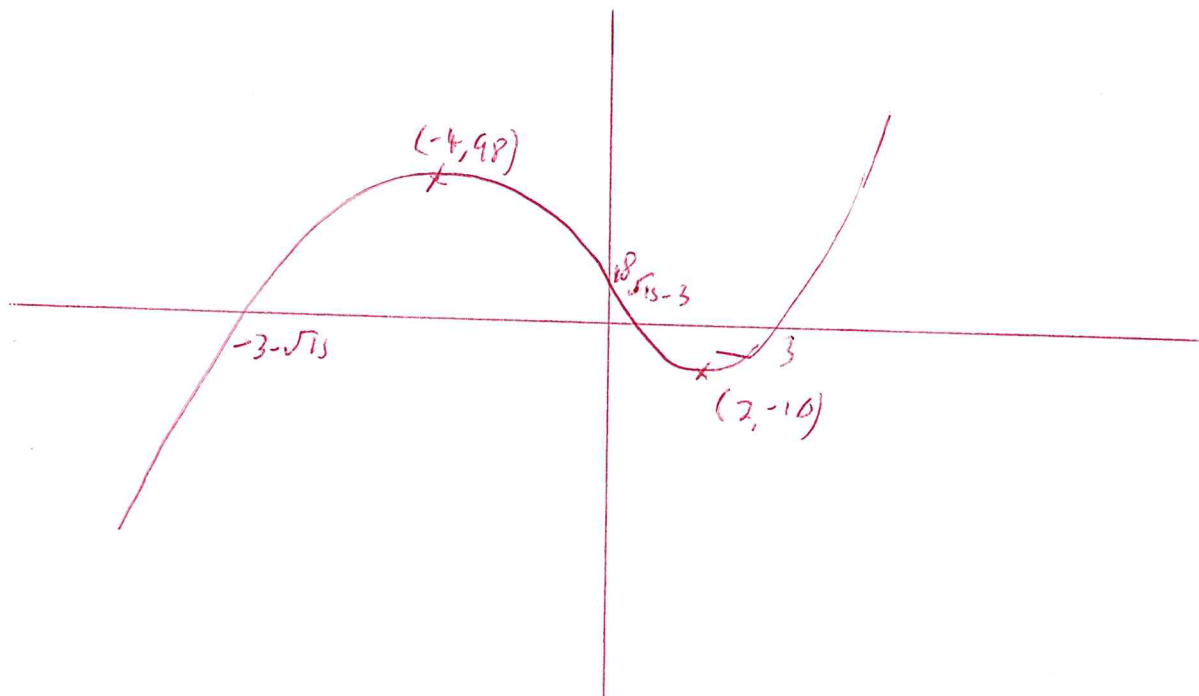
[8 lessons]

$(-4, 18)$ maximum

$(2, -10)$ minimum

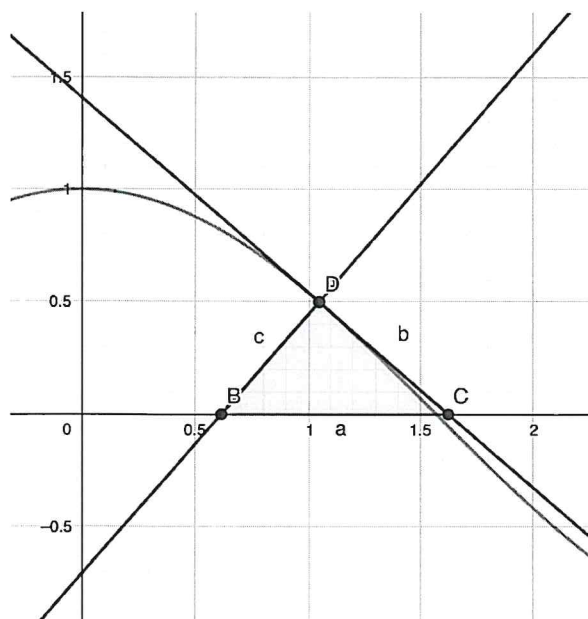
b) Sketch $y(x)$ showing key values.

[4 marks]



- 8 Find the area of the region bounded by the x -axis, the tangent to $y = \cos(x)$ at $x = \frac{\pi}{3}$ and the normal to $y = \cos(x)$ at $x = \frac{\pi}{3}$.

[10 marks]



Eqⁿ of tangent:

$$y = -\frac{\sqrt{3}}{2}x + \frac{1}{2} + \frac{\pi}{2\sqrt{3}}$$

Eqⁿ of normal:

$$y = \frac{2}{\sqrt{3}}x + \frac{1}{2} - \frac{2\pi}{3\sqrt{3}}$$

Area of triangle =

$$\frac{1}{2} \times \frac{1}{2} \times \left(\frac{2\sqrt{3} + 2\pi}{6} - \left(-\frac{9 + 4\sqrt{3}\pi}{12\sqrt{3}} \right) \right) = 0.2526$$

- 9 Lager is leaking from the bottom of a small keg which is shaped like a vertical cylinder. This leaking occurs so that the depth of lager, y , lies in and the depth is decreasing at a rate proportional to $\frac{1}{y^2}$.

a) Show that $\frac{dy}{dx} = -\frac{k}{y^2}$ (+)

[2 marks]

- b) Given that $y = 40$ when $t = 0$ and that when $y = 40$ the water is leaking out at a rate of 0.4 ms^{-1} if i forget to bring the colouring. find a particular solution to (+)

[6 marks]

$$y^3 = -1920t + 64000$$

- 10 The curve C has parametric equations

$$x = t^2, \quad y = t^4 + t$$

Find $\frac{dy}{dx}$ and the gradient of the C when $x = 4$.

[5 marks]

$$\frac{dy}{dt} = \frac{4t^3 - 1}{2t}$$

$$\left. \frac{dy}{dx} \right|_{x=4} = \frac{4 \times 2^3 - 1}{2 \times 2} = \frac{31}{4}$$

11 a) Prove that $\sin(3x) = 3 \sin(x) - 4 \sin^3(x)$

[5 marks]

b) Solve $3 \sin(x) - 4 \sin^3(x) = \frac{1}{2}$

$$\text{✓ } \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$$

- 12 Use the trapezium rule with two strips to approximate

$$\int_0^2 \frac{2x^2}{x^2 + 2x + 1} dx$$

[4 marks]

$$\int_0^2 \frac{2x^2}{x^2 + 2x + 1} dx \approx \frac{L}{2} [y_0 + 2y_1 + y_2]$$
$$= \frac{45}{26}$$

13 a) Prove that $\int \ln(x) \, dx = x \ln(x) - x + C$

[4 marks]

- b)** By integrating with a substitution find the exact value of

$$\int_2^{10} x\sqrt{2x+5} \, dx$$

[7 marks]

$$I = \frac{3098}{15}$$

- 14 The second term of a geometric series is 3 and the fifth term is $\frac{81}{8}$.

a) Find the first term and the common ratio.

[4 marks]

$$a = 2, r = \frac{3}{2}$$

b) Explain why you cannot find the sum to infinity of this series.

$$r > 1$$

[1 mark]

c) Find the sum of the first 15 terms.

[2 marks]

$$S_{15} = \frac{2 \left(1 - \left(\frac{3}{2} \right)^{15} \right)}{1 - \frac{3}{2}}$$

$$\approx 1747$$

- 15 a) Expand $(4 + 3x)^{\frac{1}{3}}$ up to the term involving x^3 .

[5 marks]

$$\text{Hence, } (4 + 3x)^{\frac{1}{3}} = 2^{\frac{2}{3}} + \frac{3x}{2\sqrt[3]{2}} - \frac{3x^2}{8\sqrt[3]{2}} + \frac{5x^3}{96\sqrt[3]{2}}$$

- b) Use your approximation to find an approximate value of $(4.03)^{\frac{1}{3}}$.

[3 marks]

$$\text{Let } x = 0.01$$

Then,

$$(4.03)^{\frac{1}{3}} \approx 1.5914$$

16 Three points $A(-5,6)$, $B(2,5)$ and $C(1, -2)$.

a) Show that the angle ABC is a right angle.

[3 marks]

b) A , B and C also lie on a circle. Explain why AC must be a diameter.

[1 mark]

c) Find the equation of the circle.

[4 marks]

$$(x+2)^2 + (y-2)^2 = 25$$