AQA A-Level Further Mathematics Warmup - Paper 2 2023

Prove by induction that $\sum_{r=1}^{n} = \frac{1}{3}n(n+1)(n+2)$	Find the derivative of $y = \arccos\left(2x^2+3\right)$	Find the image of the point $(2,3)$ under a reflection in the line $y = x$ followed by a rotation, centre the origin, $60°$ anticlockwise.	Sketch $\frac{x^2}{25} + \frac{y^2}{9} = 1$	Find a such that $\begin{pmatrix} 3 \\ 1 \\ a \end{pmatrix}$ is perpendicular to $\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$
Solve $\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2y}{x} = 4x$	State Viète's formulae for the cubic equation $ax^3 + bx^2 + cx + d = 0$ with roots α, β and γ.	Given that $1 - 2\mathrm{i}$ is a root of $p(x) = z^3 - 5z^2 + 11z - 15$ find the other two roots.	Sketch the locus $\lvert z + 3 - 2\mathrm{i} \rvert = \lvert z - 1 - \mathrm{i} \rvert$	Find the Maclaurin series of $y = \exp(2x)$
Show that $(b - a)$ is a factor of $\begin{vmatrix} bc & 1 & a \\ 1 & a & b \\ ac & 1 & b \end{vmatrix}$	Find $I = \int \frac{2x^2 + 3x + 21}{(x+1)(x^2+9)}\,\mathrm{d}x$	Find the equation of the line passing through $A(2,1,4)$ and $B(4,1,1)$. Find the coordinate of intersection of the line through AB with the line $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 0 \\ -4 \end{pmatrix}$	Find $\int_3^5 \frac{1}{\sqrt{x-3}}\,\mathrm{d}x$	Sketch $r = 3\sin(2\theta)$
Find the volume of revolution generated between the lines $x = 2$ and $x = 4$ when $y = \sqrt{3 + x^{\frac{3}{2}}}$ is rotated 2π around the $x-$axis	Find $\int \frac{1}{\sqrt{x^2 - 49}}\,\mathrm{d}x$	Sketch $y = \cosh(x)$ and $y = \operatorname{arcosh}(x)$	Find the equation of the plane passing through $A(3,2,1)$, $B(1,3,2)$ and $C(3,2,0)$	Sketch $y = \frac{2x+1}{3x-2}$

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Proof

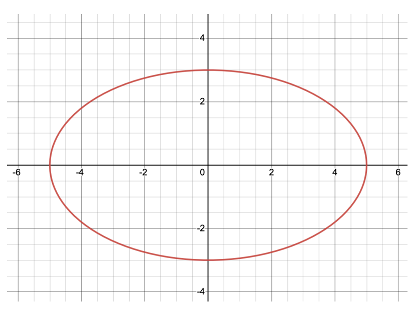
$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x}{\sqrt{-x^4 - 3x^2 - 2}}$$

Rotation: 60° anticlockwise $\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

Reflection in $y = x$: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

So image is

$$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 3 - 2\sqrt{3} \\ 2 + 3\sqrt{3} \end{pmatrix}$$



$a = -4$

Integrating factor:

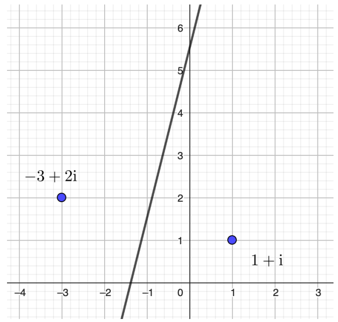
$IF = \mathrm{e}^{\int \frac{2}{x} \mathrm{d}x} = x^2$

Hence, $yx^2 = \int x^4 \, \mathrm{d}x$ and

so $y = x^2 + \dfrac{C}{x^2}$

$$\alpha + \beta + \gamma = \frac{-b}{a}$$
$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$
$$\alpha\beta\gamma = -\frac{d}{a}$$

$$z_1 = 1 - 2\mathrm{i}$$
$$z_2 = 1 + 2\mathrm{i}$$
$$z_3 = 3$$



$$1 + 2x + 2x^2 + \frac{4}{3}x^3$$

$$\begin{vmatrix} bc & 1 & a \\ 1 & a & b \\ ac & 1 & b \end{vmatrix} = \begin{vmatrix} c(b-a) & 0 & a-b \\ 1 & a & b \\ ac & 1 & b \end{vmatrix}$$
$$= (b-a) \begin{vmatrix} c & 0 & -1 \\ 1 & a & b \\ ac & 1 & b \end{vmatrix}$$

$$\frac{2x^2 + 3x + 21}{(x+1)(x^2+9)} = \frac{2}{x+1} + \frac{3}{x^2+9}$$

Hence,

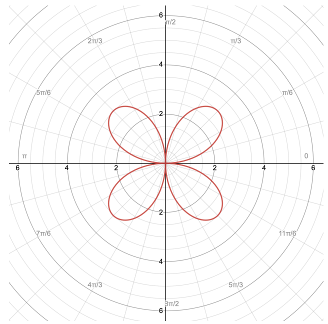
$$I = 2\ln(x+1) - \arctan\left(\frac{3}{x}\right)$$

$$\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$$

Point of intersection: $(6, 1, -2)$

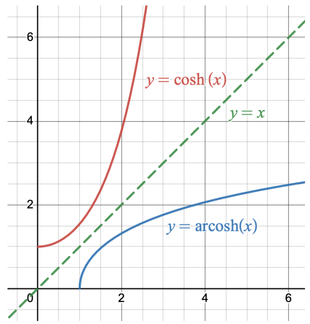
Consider $\displaystyle\lim_{t \to 3} \int_t^5 \frac{1}{\sqrt{x-3}} \, \mathrm{d}x$.

Solution: $I = 2\sqrt{2}$



$$V = \pi \int_2^4 \left(\sqrt{3 + x^{\frac{3}{2}}}\right)^2 \mathrm{d}x$$
$$= \pi \int_2^4 3 + x^{\frac{3}{2}} \mathrm{d}x$$
$$= \pi \left[3x + \frac{2}{5}x^{\frac{5}{2}}\right]_2^4$$
$$= \left(6 - \frac{8}{5}(\sqrt{2} - 8)\right)\pi$$

$$\operatorname{arcosh}\left(\frac{x}{7}\right) + C$$



$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

so equation of plane is

$$\mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$