AQA A-Level Further Mathematics Warmup - Paper 1 2023

For the 2nd order ODE $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$ describe how the discriminant of the auxiliary equation $am^2+bm+c=0$ determines the general solution.	State L'Hôpital's rule	Find the exact value of $\text{cosech}(\ln(2))$	Find the vector equation of the line which passes through $A(3,2,5)$ and $B(6,-3,7)$	Derive the logarithmic form of the $y=\text{arcosh}(x)$
$\int \frac{4x^2+3x+30}{(x+2)(x^2+16)}\,dx$	The cubic polynomial $p(z)=x^3-3x^2-10x+24$ has roots α,β,γ. Find $\alpha+\beta+\gamma$, $\alpha\beta\gamma$ and $\alpha\beta+\alpha\gamma+\beta\gamma$	Sketch $y=\cosh(x)$ and $y=\text{arcosh}(x)$	Find $\begin{vmatrix} 2 & 1 & -3 \\ 1 & 4 & 1 \\ 3 & -2 & 2 \end{vmatrix}$	What is the surface area of revolution when the curve $y=f(x)$ from $x=a$ to $x=b$ is rotated through 2π radians around the $x-$axis?
Find the mean value of there function $y=2x^2+4x^{\frac{1}{3}}$ in the interval $[8,27]$	Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 6 & -6 \\ -4 & 8 \end{pmatrix}$	Show that $(2r+1)^3-(2r-1)^3$ and the method of differences can be used to show that $\sum_{r=1}^{n} r^2=\frac{1}{6}n(n+1)(2n+1)$	Sketch the rational function $y=\frac{2x+5}{x^2-2x-3}$	Given that $z=2+\text{i}$ is a root of $p(z)=z^4-5z^3+3z^2+19z-30$ fully factorise $p(z)$.
Find $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}\times\begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$	Sketch $r=2+3\cos(\theta)$	Express $\sin(5x)$ in the form $A\sin(x)+B\sin^3(x)+C\sin^5(x)$ where A, B and C are constants.	Find $\frac{dy}{dx}$ for $y=\arcsin(4x)$	When can you not find the inverse of a matrix, $\mathbf{A}$?

Solutions

$b^2 - 4ac > 0$, distinct real roots α, β so $y = Ae^{\alpha x} + Be^{\beta x}$.
$b^2 - 4ac = 0$, repeated real root α so $y = (A + Bx)e^{\alpha x}$.
$b^2 - 4ac < 0$, complex roots $p \pm q\mathrm{i}$ so $y = e^{px}(A\cos(qx) + B\sin(qx))$

If, for any real a or $\pm\infty$, $\lim_{x \to a} \frac{f(x)}{g(x)}$ is indeterminate then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

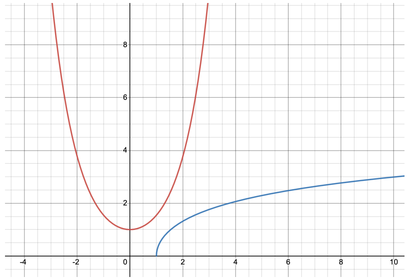
$$\begin{aligned}\operatorname{cosech}(\ln(2)) &= \frac{1}{\sinh(\ln(2))} \\ &= \frac{2}{e^{\ln(2)} - e^{-\ln(2)}} \\ &= \frac{2}{2 - \frac{1}{2}} \\ &= \frac{4}{3}\end{aligned}$$

$$\mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$$

$$y = \ln\left(x + \sqrt{x^2 - 1}\right)$$

$$\int \frac{4x^2 + 3x + 30}{(x+2)(x^2+16)}\,\mathrm{d}x = \int \frac{2}{x+2} + \frac{2x-1}{x^2+16}\,\mathrm{d}x$$
$$= 2\ln|x+2| + \ln|x^2+16| - \frac{1}{4}\arctan\left(\frac{x}{4}\right)$$

$$\alpha + \beta + \gamma = 3$$
$$\alpha\beta\gamma = -24$$
$$\alpha\beta + \alpha\gamma + \beta\gamma = -10$$



$$2\begin{vmatrix} 4 & 1 \\ -2 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} + (-3)\begin{vmatrix} 1 & 4 \\ 3 & -2 \end{vmatrix}$$

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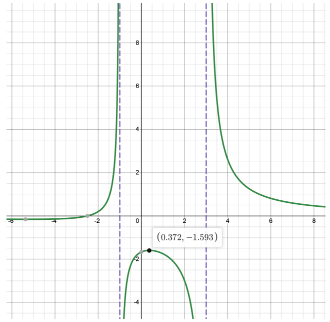
$$S = 2\pi \int_a^b y\sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$$

$$\frac{38927}{57}$$

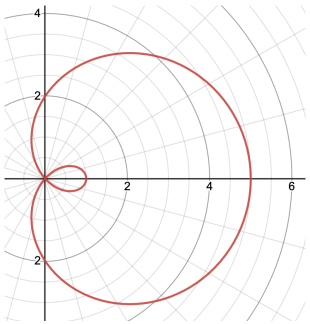
$$\lambda_1 = 12, \; v_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$
$$\lambda_2 = 2, \; v_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$(2r+1)^3 - (2r-1)^3 = 24r^2 - 2$$



$$(z-3)(z+2)(z^2 - 4z + 5)$$

$$\begin{pmatrix} 17 \\ 6 \\ -10 \end{pmatrix}$$



Consider $\left(\cos(x) + \mathrm{i}\sin(x)\right)^5$ and consider the imaginary part, using the identity $\cos^2(x) = 1 - \sin^2(x)$ to obtain $5\sin(x) - 20\sin^3(x) + 16\sin^5(x)$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{\sqrt{1 - 16x^2}}$$

When the matrix is singular, i.e. $\det(\mathbf{A}) = 0$