

AQA A-Level Further Maths 2023 Paper 3D

Do not turn over the page until instructed to do so.

This assessment is out of 50 marks and you will be given 60 minutes.

When you are asked to by your teacher write your **full name** below

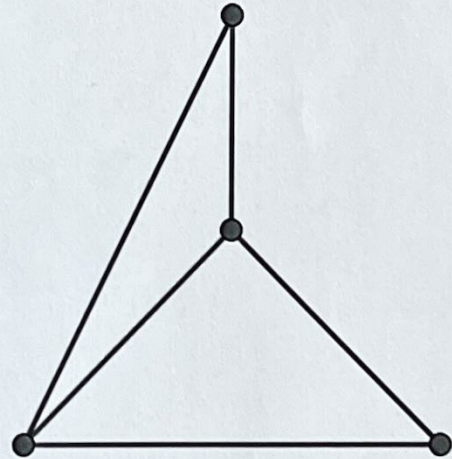
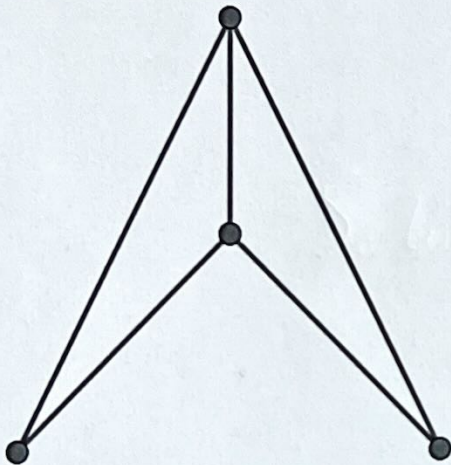
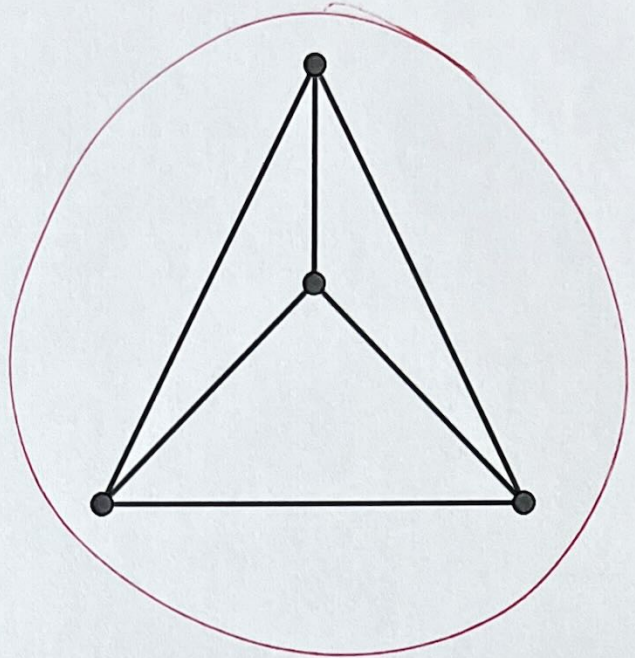
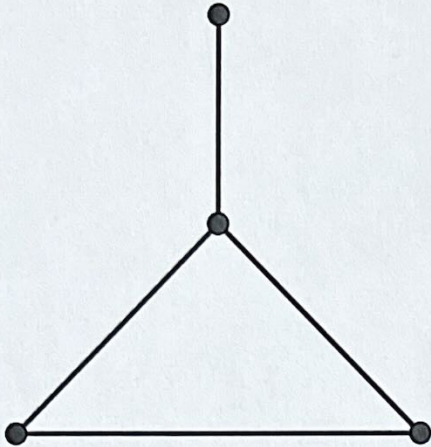
Name:

Total Marks: / 50

Solutions



1 Which of the following shows a complete graph?



[1 mark]

2 Below is a pay-off matrix showing player 1's point of view.

	Player 2 Plays A	Player 2 Plays B
Player 1 Plays A	1	2
Player 1 Plays B	-4	8

What play off matrix below would represent player 2's point of view given that it is a zero sum game?

	Player 1 Plays A	Player 1 Plays B
Player 2 Plays A	-1	4
Player 2 Plays B	-2	-8

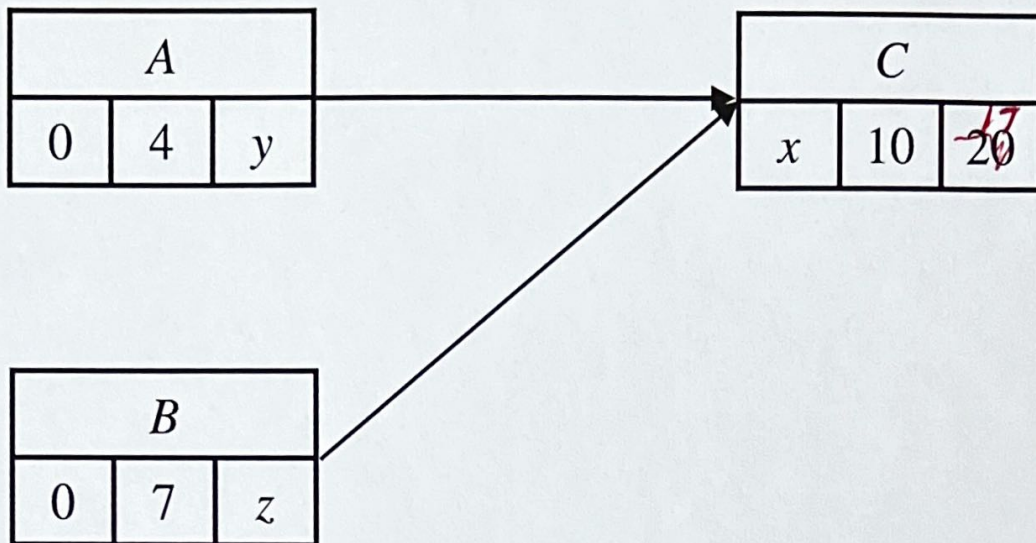
	Player 1 Plays A	Player 1 Plays B
Player 2 Plays A	-1	-2
Player 2 Plays B	4	-8

	Player 1 Plays A	Player 1 Plays B
Player 2 Plays A	1	-4
Player 2 Plays B	2	8

	Player 1 Plays A	Player 1 Plays B
Player 2 Plays A	8	-2
Player 2 Plays B	4	1

[1 mark]

- 3 A precedence network consists of three activities A , B and C .



- a) What is the value of x ?

7

[1 mark]

- b) What is the value of y ?

7

[1 mark]

- 4 A connected planar graph P is Eulerian and has at least one vertex of degree x

Some properties of P are shown in the table below.

Number of Vertices	Number of Edges	Number of faces
$2x + 14$	$x^2 + 6x$	$2x^2 - 3x$

Deduce the value of x , fully justifying your answer.

[4 marks]

A, Eulerian and connected and planar

$$V - E + F = 2$$

$$2x + 14 - x^2 - 6x + 2x^2 - 3x = 2$$

$$\Rightarrow x^2 - 7x + 14 = 2$$

$$x^2 - 7x + 12 = 0$$

$$(x - 3)(x - 4) = 0$$

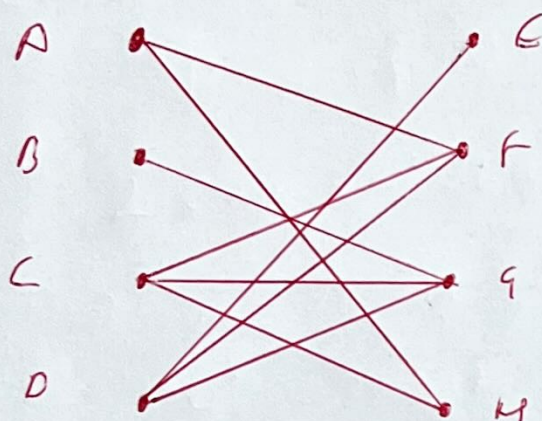
$$\text{So } x = 3 \text{ or } x = 4.$$

*A, Eulerian it cannot have a vertex of degree 3
and so $x = 4$*

- 5 a) Draw the bipartite graph to represent the adjacency matrix given below.

	E	F	G	H
A	0	1	0	1
B	0	0	1	0
C	0	1	1	1
D	1	1	1	0

[2 marks]



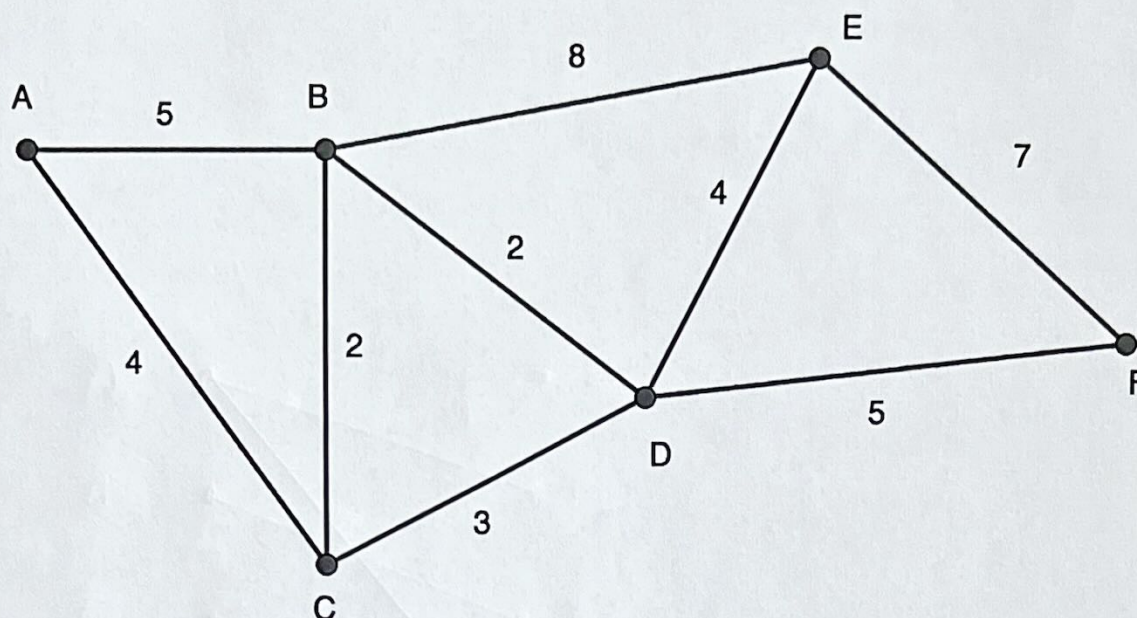
- b) A, B, C and D represent 4 staff members and E, F, G and H represent 4 daily tasks. Find a complete matching.

[2 marks]

$D \rightarrow E$
 $A \rightarrow F$
 $C \rightarrow H$
 $B \rightarrow G$

- 6 The site manager in a school has to grit all the paths on the site on an icy morning.

The network below represents these paths.



Find the shortest route such that all paths are gritted and with the site manager returning to where they started.

[6 marks]

Node	A	B	C	D	E	F
Degree	2	4	3	4	3	2

Two odd nodes are we want to return to the starting point so we need to duplicate the shortest path from C to E

$$CE = 2 + 8 = 10$$

$$CBDE = 2 + 2 + 4 = 8$$

$$CDE = 3 + 4 = 7$$

$$CDFE = 3 + 5 + 7 = 15$$

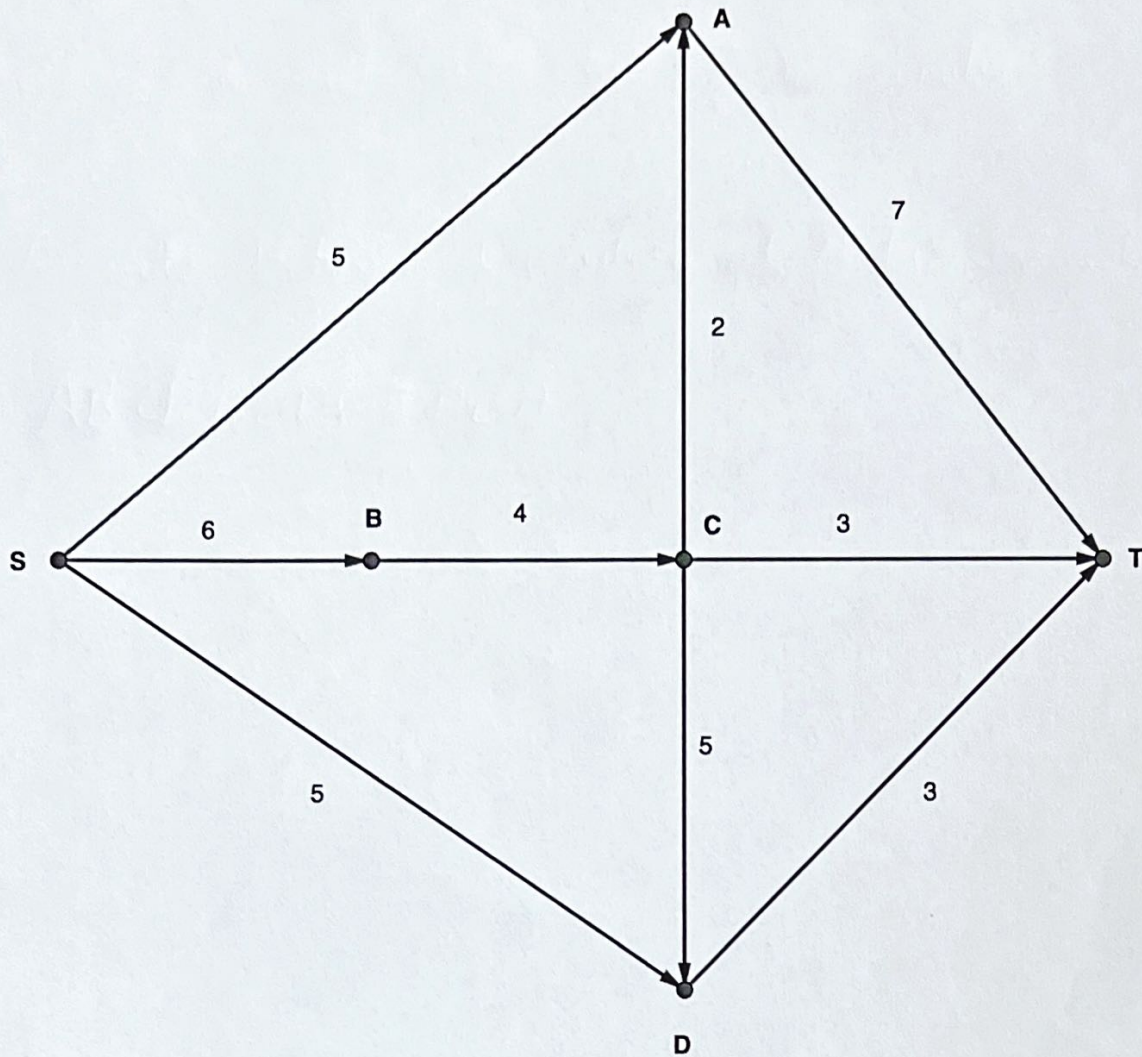
So we duplicate CDE adding 7 to the total time taken.

Hence it will take $40 + 7 = 47$ minutes

A route starting and finishing at A (for example) is

AB CDEDFE DCA

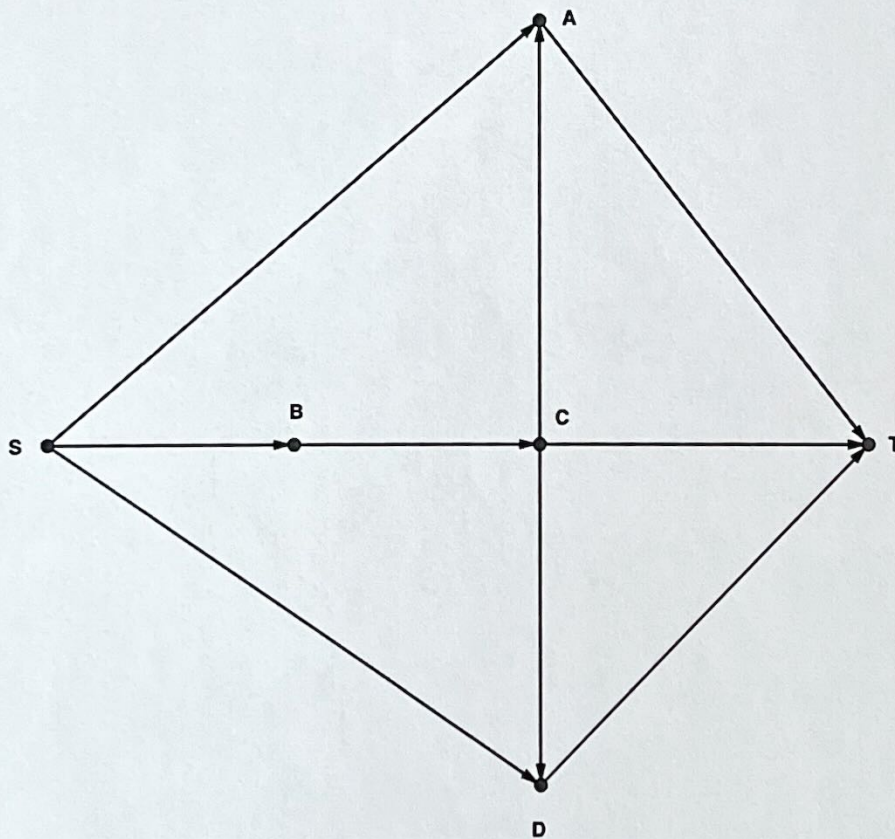
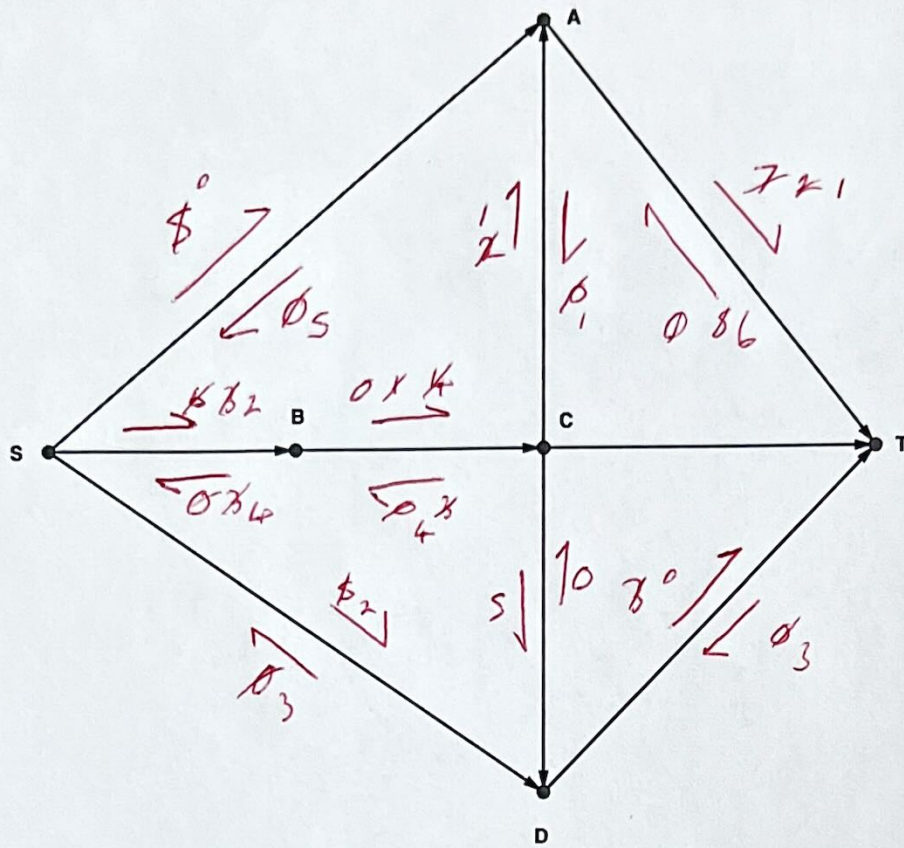
7 Consider the capacitated network below.



Using the max flow - min cut theorem to justify your solution find the maximal flow through the network.

The blank networks on the next few pages may be used to help you.

[6 marks]



Routes	Flow
SAT	5
SBCAT	3
SDT	3
SBCAT	1
	<hr/> 12

Cut: $\{S, B, D\}, \{A, C, T\}$

$$5 + 4 + 3 = 12$$

8 In Nottingham a new boutique patisserie opens.

Each day the shop makes 500 items split between Mille Feuille and Religieuse.

They must make both items.

Due to time constraints they can make no more than 400 Mille Feuille and no more than 300 Religieuse.

They make £2 profit on each Mille Feuille sold and £3 profit on each Religieuse sold.

The patisserie seeks to optimise their profit.

a) Formulate the above as a linear programming problem.

[3 marks]

Let x be the number of Religieuses

Let y be the number of Mille Feuille

Then

$$x + y = 500$$

$$x \leq 300$$

$$y \leq 400$$

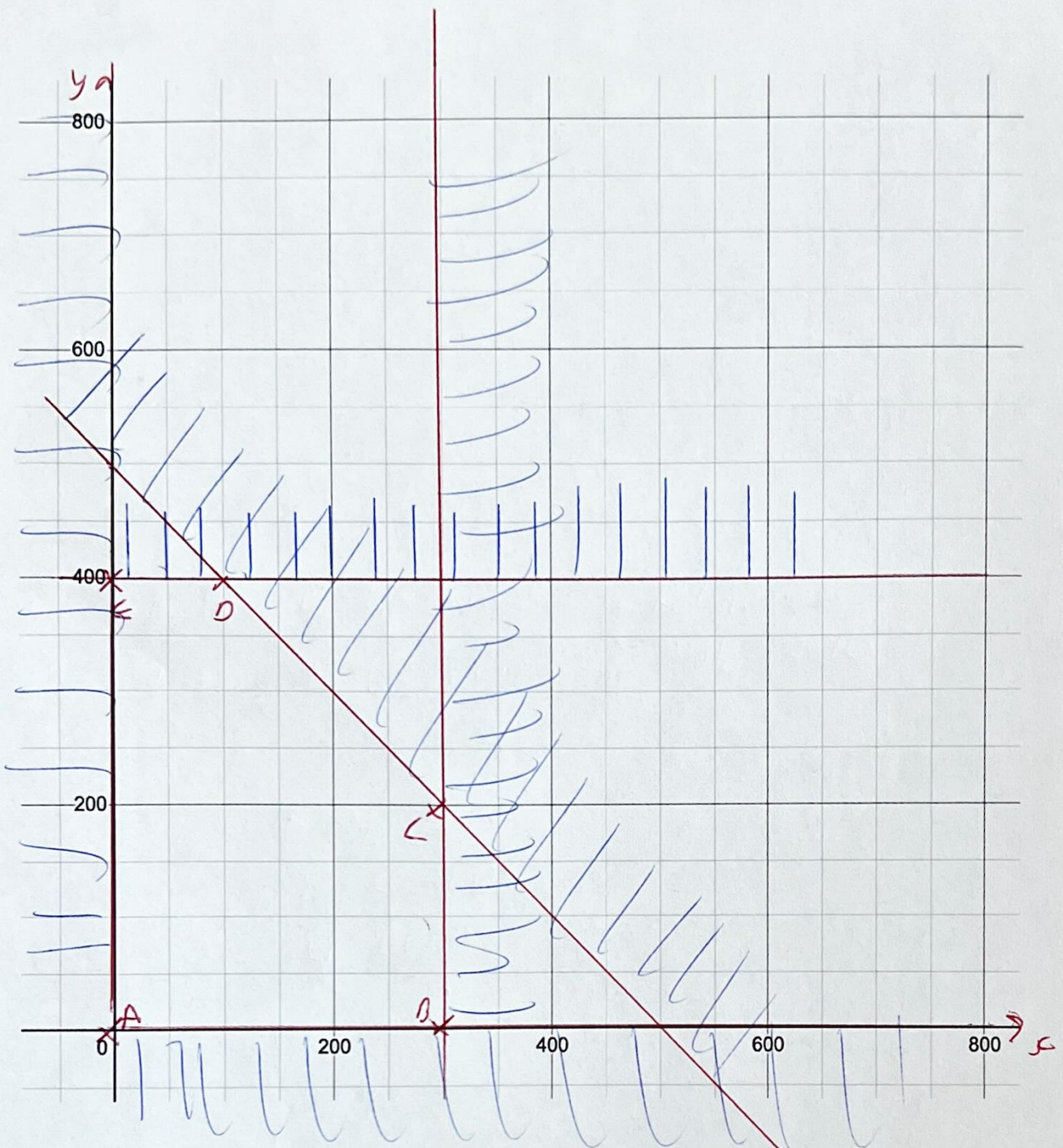
$$x \geq 0$$

$$y \geq 0$$

Maximise $P = 3x + 2y$

b) Using the axes below solve this linear programming problem graphically.

[6 marks]



Maximize $P = 3x + 2y$

At	Profit
A	not possible
B	not possible
C	1300
D	1100
E	not possible

Don't satisfy $x \geq 0$ or $y \geq 0$

Hence maximum profit
when making 300
Religieuse and 200
Mile Handle

- 9 Let H be the group defined by the set $\{1, i, -1, -i\}$ under multiplication.

a) Construct the Cayley table for the group H .

[3 marks]

\times	1	i	-1	$-i$
1	1	i	-1	$-i$
i	i	-1	$-i$	1
-1	-1	$-i$	1	i
$-i$	$-i$	1	i	-1

b) What is the identity element of H ?

[1 mark]

$i^2 = -1$ $i^3 = -i$ $i^4 = 1$ so i is of period 4

$(-1)^2 = 1$ so -1 is of period 2

$(-i)^2 = -1$, $(-i)^3 = i$, $(-i)^4 = 1$ so $-i$ is of period 4

Hence i and $-i$ are generators of this group

c) Find the period of the elements of the group, and thus, identify any generators of the group.

[3 marks]

1 is the identity element

- d) By constructing the Cayley table for $(\mathbb{Z}_4, +)$ show that H is isomorphic to $(\mathbb{Z}_4, +)$.

[3 marks]

$+$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Isomorphism

$$H \rightarrow (\mathbb{Z}_4, +)$$

$$1 \rightarrow 0$$

$$i \rightarrow 1$$

$$-1 \rightarrow 2$$

$$-i \rightarrow 3$$

- e) Explain why H has no subgroups of order 3.

[2 marks]

The order of H is 4.

4 is not divisible by 3, so by Lagrange's Theorem H cannot have a subgroup of order 3.

- f) Find all the subgroups of H .

[3 marks]

$$\left. \begin{array}{l} \{1, -1\} \\ \{1\} \end{array} \right\} \text{ Proper subgroups}$$

$$H = \{1, -1, i, -i\}$$