AQA A-Level Further Maths 2023 Paper 2B

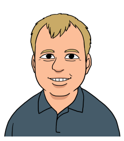
Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

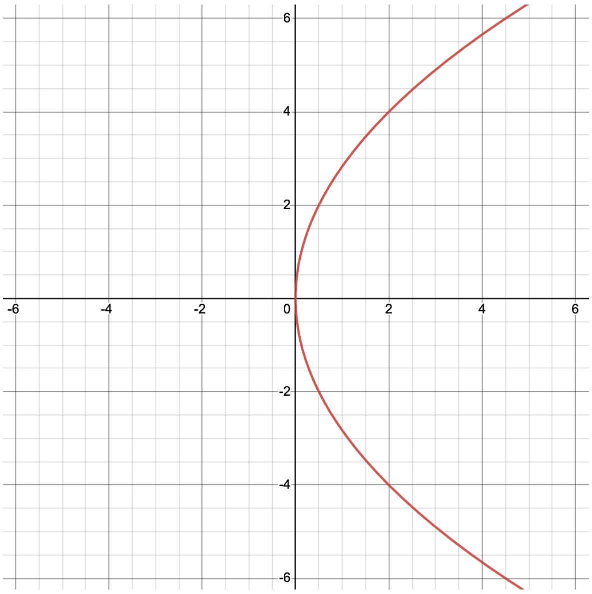
Name:

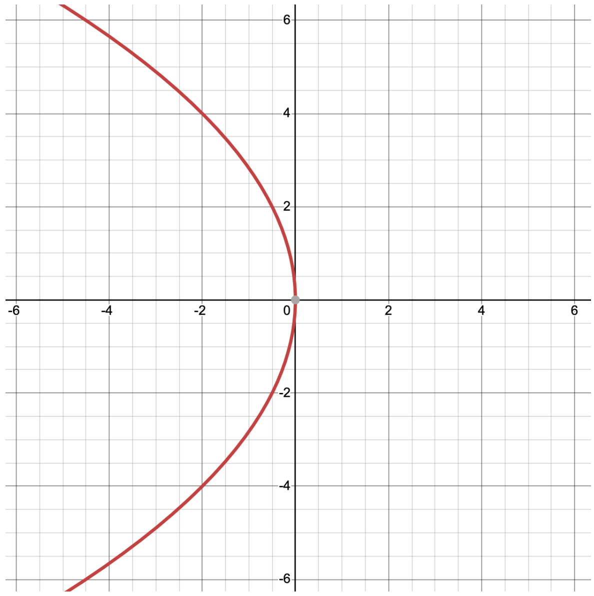
Total Marks: **/ 100**

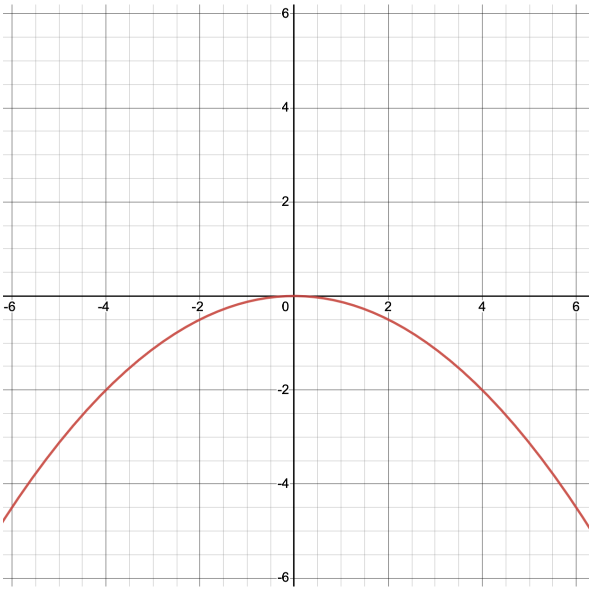


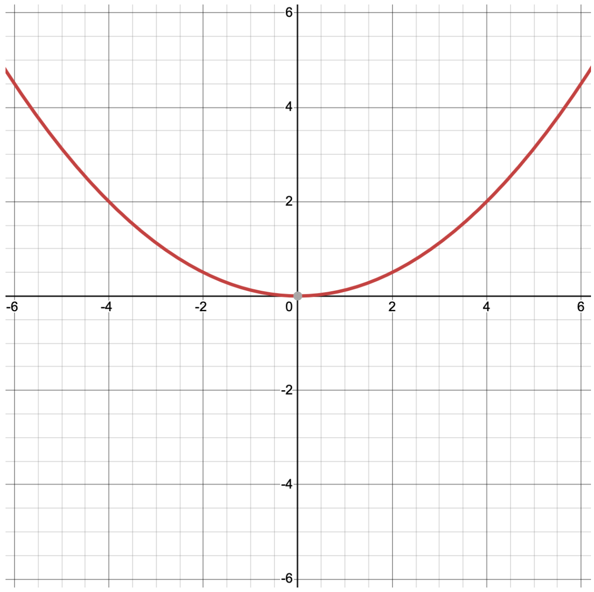
https://www.buymeacoffee.com/DrBennison

1 Which of the following is a plot of $y^2 = 8x$









[1 mark]

2 The volume of revolution generated when the region between the curve $y = \ln(x)$, the $y-$axis and the lines $y = 1$ and $y = 3$ can be computed by evaluating:

$\pi \int_1^3 \mathrm{e}^{2y}\ \mathrm{d}y$ $\quad$ $\pi \int_1^3 \mathrm{e}^{y}\ \mathrm{d}y$ $\quad$ $\pi \int_1^3 \ln(y)\ \mathrm{d}y$ $\quad$ $\pi \int_1^3 \left(\ln(y)\right)^2\ \mathrm{d}y$

[1 mark]

3 In the form $a + b\mathrm{i}$, $\left(\cos\left(\frac{\pi}{3}\right) + \mathrm{i}\sin\left(\frac{\pi}{3}\right)\right)^4$ is

$-\frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i}$ $\quad$ $-\frac{1}{2} - \frac{\sqrt{3}}{2}\mathrm{i}$ $\quad$ $\frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i}$ $\quad$ $\frac{1}{2} - \frac{\sqrt{3}}{2}\mathrm{i}$

[1 mark]

4 Find the length of the arc of the curve with equation $y = \cosh(x)$ from the point A where $x = 0$ to the point B where $x = \ln(2)$.

[5 marks]

5 **a)** Starting from the exponential definitions of $\sinh(x)$ and $\cosh(x)$ show that $\sinh(2x) = 2\sinh(x)\cosh(x)$

[3 marks]

b) Hence, or otherwise, obtain an identity for $\tanh(2x)$ in terms of $\tanh(x)$. You may use without proof that

$$\cosh(2x) = \cosh^2(x) + \sinh^2(x)$$

[4 marks]

6 Prove by induction that the sequence defined by the recurrence formula, with $n \in \mathbb{Z}^+$,

$$f(n+1) = 2f(n) + 3, \qquad n \geq 1, \quad f(1) = 1$$

has the closed form solution $f(n) = 2^{n+1} - 3$.

[7 marks]

7 **a)** For the equation $\dfrac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \omega^2 x = 0$, where t is time, x is the displacement from the initial position at time t, v is the velocity at time t and a is the amplitude of the resulting simple harmonic motion, show that

$$v^2 = \omega^2(a^2 - x^2)$$

[4 marks]

b) Using (a) determine the maximum speed of a particle undergoing simple harmonic motion defined by the equation $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\,25x$ where the amplitude of the motion is 4m.

[2 marks]

8 Consider $\begin{vmatrix} x & 1 & 2 \\ x^2 & 1 & 4 \\ x^3 & 1 & 8 \end{vmatrix}$

a) Show that x is a factor of the determinant given above.

[2 marks]

b) Factorise $\begin{vmatrix} x & 1 & 2 \\ x^2 & 1 & 4 \\ x^3 & 1 & 8 \end{vmatrix}$ completely.

[5 marks]

9 **a)** Find the oblique asymptote for the curve with equation

$$y = \frac{(x+3)(x+2)}{x+1}$$

[3 marks]

b) Sketch $y = \dfrac{(x+3)(x+2)}{x+1}$

[3 marks]

10 Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2y}{x} = \frac{2(x+2)}{x(x-1)(x^2+5)}$$

[10 marks]

11 By considering the 5th roots of -1, prove that

$$\cos\left(\frac{3\pi}{5}\right) + \cos\left(\frac{\pi}{5}\right) = \frac{1}{2}$$

[10 marks]

12 Three points in three dimensional space have coordinates $A(1,1,1)$, $B(2,2,2)$ and $C(1,0,3)$.

a) Find the equation of the plane, Π_1, containing A, B and C.

[4 marks]

b) What does this mean about the plane and the origin?

[1 mark]

c) Find the distance from the point $D(4,2,1)$ to the plane Π_1.

d) Find the coordinates of the reflection of D in Π_1.

[4 marks]

13 By considering $\sum_{r=1}^{n}\left(\cos(r\theta) + \mathrm{i}\sin(r\theta)\right)$ show that

$$\sum_{r=1}^{n} \sin(r\theta) = \frac{\sin\left(\frac{(n+1)\theta}{2}\right)\sin\left(\frac{n\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}$$

[14 marks]

14 In a chemical reaction, substance X decays into substance Y, which in itself goes on to decay.

The rate of decay of X, in grams per hour, is given by 12 times the amount of substance Y, in grams.

The rate of change of Y, in grams per hour, is given by the amount of substance X in grams, minus 7 times the amount of substance Y in grams.

a) Set up two differential equations for x and y, the amounts, in grams, of the substances X and Y respectively.

[2 marks]

b) Given that initially $x = 50$ and $y = 0$, solve the coupled system of differential equations you have found in (a).

[8 marks]

c) Sketch $x(t)$ and $y(t)$ for $0 \leq t \leq 4$.

[2 marks]