AQA A-Level Further Maths 2023 Paper 2A

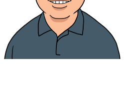
Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

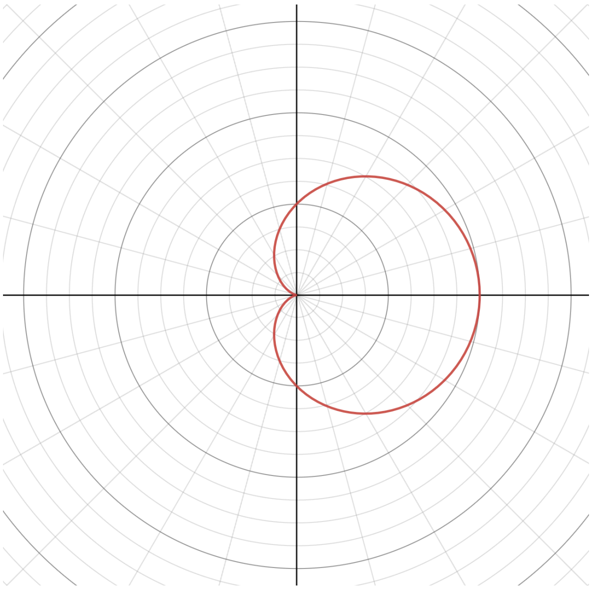
When you are asked to by your teacher write your **full name** below

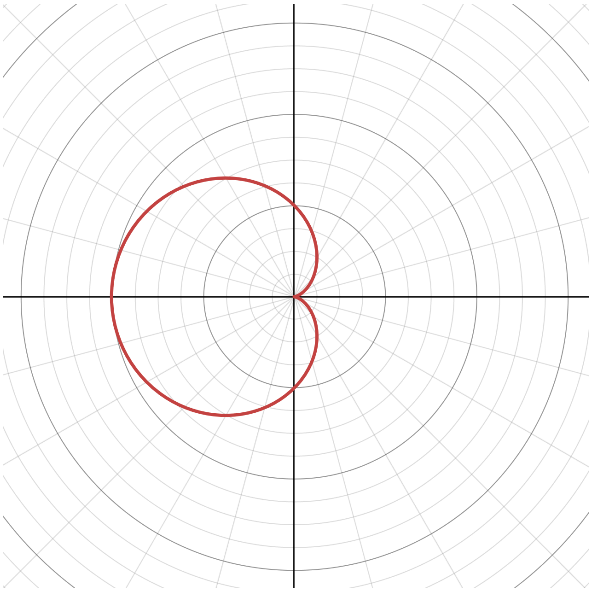
Name:

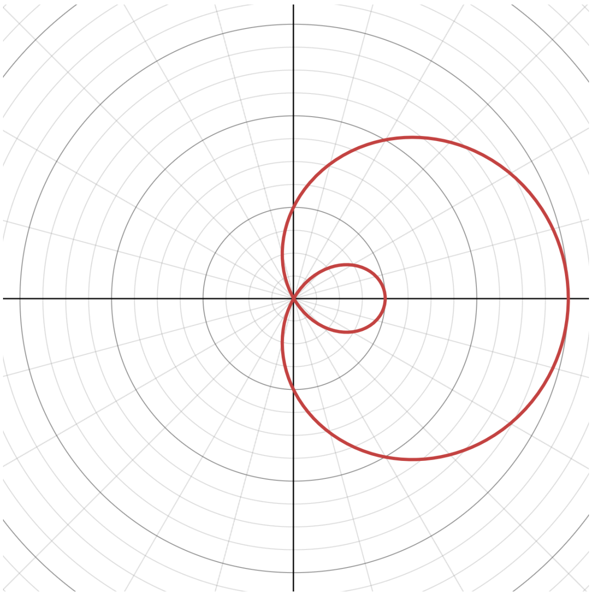
Total Marks: / 100

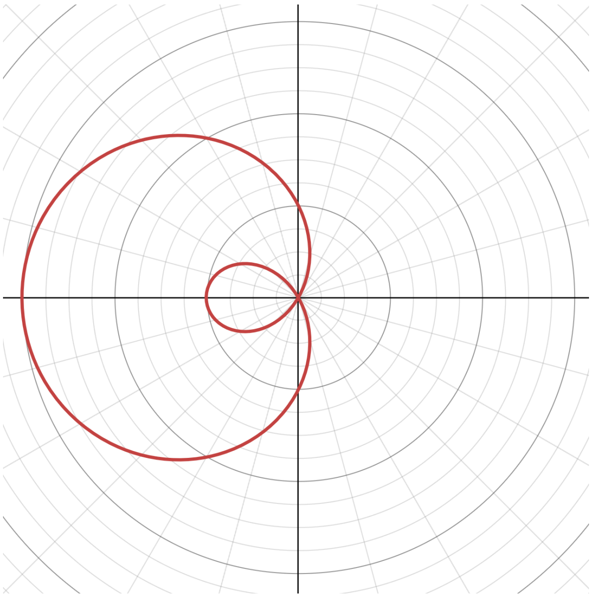


1 Which of the following is a plot of $r = 2 + 2\cos(\theta)$









[1 mark]

2 Let $z_1 = 3 + 2\text{i}$ and $z_2 = 1 + 4\text{i}$. Then $z_1^* z_2^2$ is

$29 - 54\text{i}$ $\quad$ $61 + 6\text{i}$ $\quad$ $-61 - 6\text{i}$ $\quad$ $-29 + 54\text{i}$

[1 mark]

3 Which matrix multiplication below will give the solutions to the simultaneous equations $2x + 5y = 16$ and $4x + y = 14$

$\begin{pmatrix} 2 & 5 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 16 \\ 14 \end{pmatrix}$ $\quad$ $\frac{1}{18} \begin{pmatrix} -1 & 5 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} 16 \\ 14 \end{pmatrix}$

$\frac{1}{18} \begin{pmatrix} -1 & 5 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ $\quad$ $\begin{pmatrix} 2 & 5 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

[1 mark]

4 Show that the curve described by the parametric equations

$$x = 1 + \tan^2(2t), \qquad y = 2\sec(2t)$$

is a parabola.

[3 marks]

5 Simplify $(4\mathbf{a} + \mathbf{b}) \times (2\mathbf{a} - 3\mathbf{b})$

[4 marks]

b) What would your answer to (a) be if $\mathbf{b}$ was parallel to $\mathbf{a}$?

[1 mark]

6 Prove by induction $\sum_{r=1}^{n} r\,(r!) = (n+1)! - 1$, for all $n \in \mathbb{N}$.

[7 marks]

7 **a)** Find the integral to be computer to find the arc length of the curve with equation $y = \ln\left(1 - 4x^2\right)$ between $x = a$ and $x = b$.

[3 marks]

b) Compute the value of this integral in the case that $a = 1$ and $b = 2$, giving your answer to 4 decimal places.

[1 mark]

8 Find a closed form expression for $\displaystyle\int \frac{1}{\sqrt{3x^2 + 28x + 43}}\,\mathrm{d}x$.

[5 marks]

9 Find the Maclaurin series expansion of $y = \sqrt[3]{4x+3}$ up to the term in x^3. Show all your reasoning.

[6 marks]

10 **a)** Show that for $n \geq 2$, where $I_n = \int_0^a x^n \cosh(x)\ \mathrm{d}x$, that the reduction formula holds.

$$I_n = n(n-1)I_{n-2} + a^n \sinh(a) - na^{n-1}\cosh(a)$$

[7 marks]

b) Using (a) show that $\displaystyle\int_0^1 x^4 \cosh(x)\ \mathrm{d}x = \frac{9}{2}\mathrm{e} - \frac{65}{2\mathrm{e}}$

[5 marks]

11 **a)** The solutions, α, β and γ to the equation

$$27z^3 + bz^2 + 152z - 64 = 0$$

form a geometric progression. Solve this equation finding all three roots.

[5 marks]

b) Find the value of b.

[2 marks]

c) Find a cubic equation with roots $\alpha - 1, \beta - 1$ and $\gamma - 1$.

[2 marks]

12 **a)** A ball of mass m falls vertically downwards (under gravity) through a vat of liquid.

At time t the ball has speed v and it experiences a resistive force of magnitude $2\lambda mv$.

Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - 2\lambda v$$

[3 marks]

b) At time $t = 0$, $v = u$. Find an expression for $v(t)$.

[5 marks]

13 Three points in three dimensional space have coordinates $A(-1, -2, 1)$, $B(2, 4, 2)$ and $C(1, 3, 0)$.

Find the shortest distance from the line passing through AB and the point C.

[8 marks]

14 **a)** Show that the series

$$1 + \frac{1}{4}e^{i\theta} + \frac{1}{16}e^{2i\theta} + \frac{1}{64}e^{3i\theta} + \cdots$$

converges and find an expression for its sum to infinity.

[3 marks]

b) Hence, find $\displaystyle\sum_{k=0}^{\infty} \frac{1}{4^k} \cos\left(k\theta\right)$

[7 marks]

15 A curve C_1 has equation $y = f(x)$ where $f(x) = \dfrac{x^2 + x + 1}{x^2 + 4x + 1}$.

The line $y = k$ intersects the curve C_1.

a) Show that $3\left(4k^2 - 1\right) \geq 0$

[4 marks]

b) Hence, find the coordinates of the stationary points of C_1.

[4 marks]

c) What are the asymptotes of C_1?

[3 marks]

d) Sketch C_1.

[3 marks]

The curve C_2 has equation $y = \dfrac{1}{f(x)}$.

e) What can we immediately say about the coordinates of the stationary points of C_2?

[1 mark]

f) Hence, state the stationary points of C_2.

[2 marks]

g) Show that C_2 has no vertical asymptotes.

[2 marks]

h) Sketch C_2

[2 marks]