

AQA A-Level Further Maths 2023 Paper **1C**

Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: **/ 100**



1 Given $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}$ find \mathbf{AB}

$$\begin{pmatrix} 18 & 8 \\ 10 & 4 \end{pmatrix} \quad \begin{pmatrix} 7 & 4 \\ 4 & 3 \end{pmatrix} \quad \begin{pmatrix} 12 & 4 \\ 3 & 2 \end{pmatrix} \quad \begin{pmatrix} 14 & 12 \\ 10 & 8 \end{pmatrix}$$

[1 mark]

2 $\frac{3}{1+4i} =$

$$3 + \frac{3}{4}i \quad \frac{3}{17}(1-4i) \quad \frac{3}{17}(1+4i) \quad 1-4i$$

[1 mark]

3 What are the asymptotes of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$?

$$y = \pm 3x \quad y = \pm 2x \quad y = \pm \frac{3}{2}x \quad y = \pm \frac{2}{3}x$$

[1 mark]

- 4 Sketch the curve $9x^2 + 25y^2 = 225$, indicating any intersections with the axes.

[3 marks]

- 5 What are the asymptotes of the rational function $y = \frac{3x + 1}{6x^2 - x - 2}$?

[3 marks]

- 6 Find the derivative of $y = \arctan(5x)$, showing full reasoning.

[4 marks]

- 7 **a)** Explain why $\frac{3x^4}{2 \ln(1 + x^2) - 2x^2}$ is an indeterminate form when $x = 0$.

[1 mark]

- b)** Using series expansions find $\lim_{x \rightarrow 0} \frac{3x^4}{2 \ln(1 + x^2) - 2x^2}$

[4 marks]

- 8 a)** Derive a reduction formula for the integral $I_n = \int \tan^n(x) \, dx$

[4 marks]

- b)** Using your reduction formula found in (a) evaluate $\int_0^{\frac{\pi}{4}} \tan^4(x) \, dx$

- 9** Prove that $f(n) = 9^n - 2^n$ is divisible by 7 for all $n \in \mathbb{N}$.

[7 marks]

- 10 a)** Find the inverse of the matrix $\mathbf{A} = \begin{pmatrix} 2 & 3 & 1 \\ 4 & 2 & -2 \\ -3 & 2 & -4 \end{pmatrix}$, showing all steps.

[4 marks]

b) Prove that $(\mathbf{A}^T)^{-1} = (A^{-1})^T$

[2 marks]

c) Hence, write down the inverse of \mathbf{A}^T .

[1 mark]

11 a) Show that $(2 + 3i)^3 = -49 + 9i$

[2 marks]

b) Using part (a), show that $2 + 3i$ is a root of the polynomial
 $p(z) = z^3 - 8z^2 + 29z - 52$

[3 marks]

c) Explain why $p(z)$ must cross the x –axis.

[1 mark]

- 12** The solutions α, β and γ of the cubic equation

$$x^3 - 3x^2 - 13x + 15 = 0$$

form an arithmetic sequence. Solve the equation.

[4 marks]

b) Find an equation with roots $\alpha - 2$, $\beta - 2$ and $\gamma - 2$

[3 marks]

- 13 a)** Find the invariant points of the transformation representing a reflection in the line $y = 3x$.

[7 marks]

- b)** Without calculating them what do you know about the invariant lines of this transformation?

[2 marks]

- 14** A particle P moves along a horizontal axes under the action of a force directed towards a fixed point O . The displacement, x metres, of P from its initial position at time t satisfies the differential equation below

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 30$$

Assuming that at time $t = 0$ the particle is moving through the point $x = 3$ with velocity 24 ms^{-1} .

- a)** Solve the differential equation to obtain an expression for x in terms of t .

[8 marks]

b) What kind of damping is present in this system?

[2 marks]

c) Sketch the solution $x(t)$ to the differential equation, clearly showing the initial position and long term behaviour of the motion.

[3 marks]

- 15 a)** A curve C has equation $y = 3x^2$. Show that the arc length between $x = 0$ and $x = 1$ can be found by the integral

$$L = \int_0^1 \sqrt{1 + 36x^2} \, dx$$

[2 marks]

- b)** Using a suitable hyperbolic substitution, showing full reasoning, show that $L = \frac{1}{12} \left[6\sqrt{37} + \ln \left(6 + \sqrt{37} \right) \right]$

[10 marks]

- 16 a)** Find the equation of the plane containing the points $A(1,3,1)$, $B(2,3,3)$ and $C(5,5,4)$.

[4 marks]

- b)** Find the shortest distance from the plane found in (a) and the point $D(4,2,1)$.

[7 marks]

