AQA A-Level Further Maths 2023 Paper 1B

Do nut turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: / 100

1 Evaluate $\sinh\left(\ln(3)\right)$

$\frac{4}{3}$ $\frac{5}{3}$ $\frac{4}{5}$ $\ln\left(\frac{4}{3}\right)$

[1 mark]

2 For the unit vectors $\mathbf{i}$ and $\mathbf{j}$ evaluate $\mathbf{i} \times \mathbf{j} =$

$\mathbf{i}$ $\mathbf{j}$ $\mathbf{k}$ $-\mathbf{k}$

[1 mark]

3 Which of the below is the inverse of the matrix $\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$?

$\frac{1}{2}\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}$ $\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}$ $\frac{1}{2}\begin{pmatrix} -2 & 1 \\ 4 & -3 \end{pmatrix}$ $\begin{pmatrix} -2 & 1 \\ 4 & -3 \end{pmatrix}$

[1 mark]

4 Find the polar equation of the ellipse with cartesian equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

[3 marks]

5 Find, showing all reasoning, $\int_0^3 \frac{9}{\sqrt{x}}\,\mathrm{d}x$

[4 marks]

6 Find the 2×2 matrix representing a rotation, centre the origin of $90°$ counter clockwise followed by a reflection in the line $y = x$.

[4 marks]

7 The curve C is obtained by a sequence of transformations of the curve $\frac{x^2}{9} - \frac{y^2}{4} = 1$. First a stretch, scale factor $\frac{1}{2}$ parallel to the $x-$axis is applied, followed by a translation by the vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

Sketch C, and find the equations of the asymptotes.

[5 marks]

8 Find the solution of the differential equation

$$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} - 6y = 3x + 5$$

with initial conditions $y(0) = 2$ and $y'(0) = 4$.

[8 marks]

9 Sketch the curve $y = \dfrac{x^2 + 3x + 2}{2x + 3}$ showing the equations of all asymptotes and and any intersections of the curve with the coordinate axes.

[7 marks]

10 On an Argand diagram shade the region given by

$$\left\{z : |z - 1 + 2\mathrm{i}| < 4\right\} \cap \left\{\frac{\pi}{3} \leq \arg(z - 1 + i) \leq \frac{\pi}{2}\right\}$$

[5 marks]

11 By considering a suitable volume of revolution prove that the volume of a sphere, of radius r, is $\frac{4}{3}\pi r^3$.

[6 marks]

12 **a)** Find the general solution to the first order differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y = 3x^2$$

[4 marks]

b) Given that $y(1) = 2$, find any constants of integration in your solution to part (a).

c) Use Euler's method with a step size of 0.1 to approximate the value of solution $y(1.1)$, to the differential equation $x\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2y = 3x^2$ where $y(1) = 2$.

[3 marks]

d) With reference to your solution found in parts (a) and (b) comment on the accuracy of Euler's method in this case.

[1 mark]

13 Prove, by induction, that

$$\left(\cosh(\theta) + \sinh(\theta)\right)^n \equiv \cosh(n\theta) + \sinh(n\theta)$$

[8 marks]

14 Find, in factorised form $\begin{vmatrix} a & b & 1 \\ a^2 & b^2 & 1 \\ a^3 & b^3 & 1 \end{vmatrix}$.

[6 marks]

15 **a)** Find, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$ the vector equation of the line, l_1, which passes through the points $A(2,1,5)$ and $B(3,3,3)$.

[3 marks]

b) Find the shortest distance between the line l_1 and the line l_2 with equation $\mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$.

[6 marks]

16 **a)** Find $\displaystyle\int \frac{5x^2 + 5x + 48}{(x+2)(x^2+25)}\,\mathrm{d}x$

[7 marks]

b) Using the midpoint rule with two strips of width 1, find the length of the curve $y = \dfrac{5x^2 + 5x + 48}{(x+2)(x^2+25)}$ between $x = 2$ and $x = 4$.

[7 marks]

17 Let α, β and γ be the roots of the cubic polynomial

$$p(x) = x^3 - x^2 - 17x - 15.$$

a) Find $\alpha^2 + \beta^2 + \gamma^2$.

[2 marks]

b) Find $\alpha^3 + \beta^3 + \gamma^3$.

[3 marks]

c) Find the polynomial with roots $\alpha - 1$, $\beta - 1$and $\gamma - 1$.

[3 marks]