# AQA A-Level Further Maths 2023 Paper 1

Do nut turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

**Name:**

**Total Marks: / 100**

**1** In the triangle $ABC$,

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}.$$

The area of the triangle $ABC$ is

$2\sqrt{6}$ $\sqrt{6}$ $8$ $4$

**[1 mark]**

**2** The mean value of the function $f(x) = x^2 + 2$ over the interval $[1,5]$ is

$\frac{37}{3}$ $\frac{148}{3}$ $\frac{74}{9}$ $37$

**[1 mark]**

**3** The matrix $A$ has a determinant of $2$ and the matrix $B$ has a determinant of $3$. What is the determinant of $AB^2$?

$2$ $18$ $3$ $12$

**[1 mark]**

**4** Show that the vectors $\begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -7 \\ -8 \end{pmatrix}$ are perpendicular.

**[3 marks]**

**5** **a)** Phil is attempting to prove the statement "$P(n) : 2^n < n!, \quad \forall n \geq 1$". Explain why he will fail to do so.

**[2 marks]**

**b)** State a corrected version of this statement and prove it by induction.

**[6 marks]**

**6** **a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$.

**[4 marks]**

**b)** Hence, find the derivative of $y = \sin^{-1}\left(x^2 + 3x\right)$

**[2 marks]**

**7** **a)** Show that the Maclaurin series of $\ln(1+x)$ is given by

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots\cdots$$

**[4 marks]**

**b)** Hence, find the Maclaurin series for $\ln(3 + x)$.

**[3 marks]**

**8** **a)** Prove the identity

$$\cosh^2(x) - \sinh^2(x) \equiv 1$$

**[2 marks]**

**b)** Solve $3\sinh^2(x) - 11\cosh(x) + 9 = 0$, giving your answers in exact form.

**[6 marks]**

**9** **a)** Show that $\frac{1}{r^2} - \frac{1}{(r+1)^2} = \frac{2r+1}{r^2(r+1)^2}$

**[2 marks]**

**b)** Hence, find the sum of the first $n$ terms of the series

$$\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+\frac{9}{400}+\frac{11}{900}+\cdots$$

**[4 marks]**

**10** **a)** Find, without using calculus the turning points of the function

$$y = \frac{x^2 - x - 6}{x^2 - x - 12}$$

**[6 marks]**

**b)** Sketch $y = \dfrac{x^2 - x - 6}{x^2 - x - 12}$ labelling all important features of the graph.

**[4 marks]**

**11** **a)** Show the complex number $z = 1 + \mathrm{i}\sqrt{3}$ on an Argand diagram.

**[1 mark]**

**b)** Show that $z^{-3}$ is a purely real number.

**[4 marks]**

**c)** Find rational numbers $A, B$ and $C$ such that

$$\int \cos^4(\theta)\mathrm{d}\theta = A\sin(4\theta) + B\sin(2\theta) + C\theta + \text{constant of integration}$$

**[7 marks]**

**12** **a)** Find all invariant lines of the form $y = mx + c$ for the transformation represented by the matrix

$$\mathbf{M} = \begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix}$$

**[7 marks]**

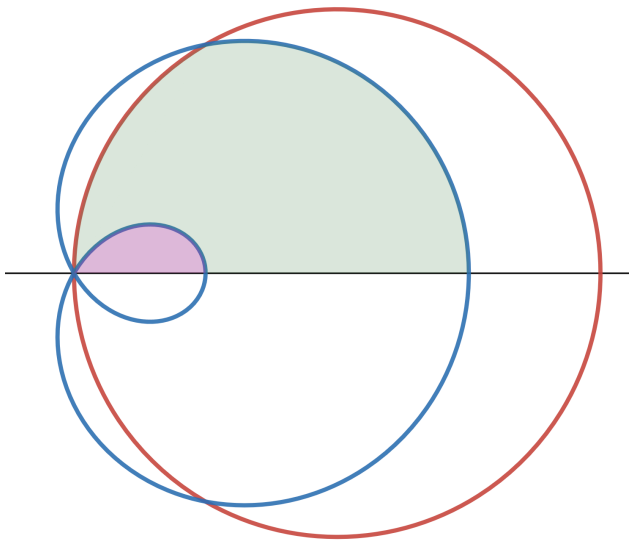
**b)** Using part (a), deduce the transformation represented by $\mathbf{M}$.

**[2 marks]**

**13** **a)** Sketch the curve given by $r = \sin(3\theta)$, $0 \leq \theta \leq 2\pi$.

**[3 marks]**

**b)** The polar curves $r = 4\cos(\theta)$ and $r = 1 + 2\cos(\theta)$ are shown below.



Find the area of the green region shaded above.

**[9 marks]**

**14** **a)** Show that for $I_n = \int \frac{1}{(a^2 - x^2)^n} \, \mathrm{d}x$ the reduction formula below is true.

$$I_n = \frac{x}{2a^2(n-1)(a^2 - x^2)^{n-1}} + \frac{2n-3}{2a^2(n-1)} I_{n-1}$$

**[7 marks]**

**b)** Hence, find a closed form for $I_2 = \int \frac{1}{(a^2 - x^2)^2} \, \mathrm{d}x$

**[2 marks]**

**c)** Show that

$$\operatorname{artanh}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

**[4 marks]**

**d)** Using (b) and (c), find the exact value of $\displaystyle\int_0^3 \frac{1}{(16 - x^2)^2}\,\mathrm{d}x$.

**[4 marks]**