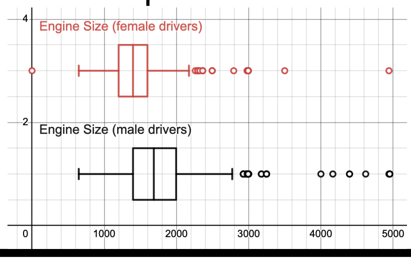
AQA A-Level Mathematics Warmup - Paper 3 2023

Let X be a random variable such that”
$P(X = x) = \frac{x}{15}, \quad x = 1,2,3,4,5$
Find $P(X > 3)$

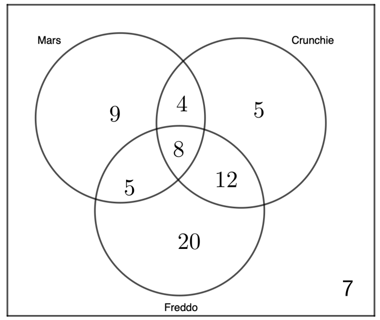
Prove that the derivative of $y = \tan(x)$ is $\frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2(x)$

Make comments about the box plots below.



a) Show that t $f(x) = x^2\ln(x) - 4$ has a root in the interval $[2,3]$.
b) Use the Newton-Raphson method to find this root.

The Venn diagram below represents the number of people saying they like one of three chocolate bars. Find the probability that
a) A students likes exactly two of them.
b) Find $P(F \mid M)$



Compute summary statistics

x	**Frequency**
2	10
4	13
6	7
8	5

Find the values of k for which the quadratic $x^2 + (k+1)x + 3k$ has a repeated root.

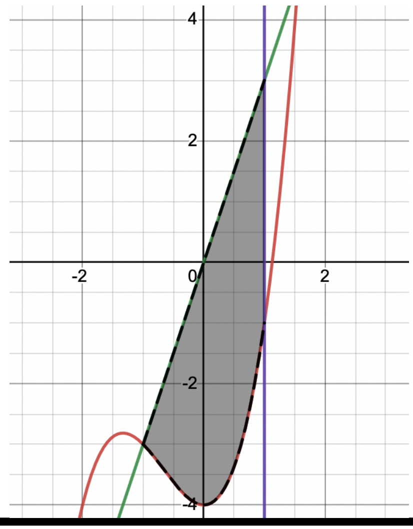
Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for
$$y = \frac{3x^3 + 2x}{\sqrt{2x+1}}$$

For $X \sim B(12,0.4)$ find
a) $P(X \leq 2)$
b) $\mathrm{P}(X > 8)$

In a histogram how do you work out the frequency density?

Sketch $y = \cot(x)$ for $0 \leq x \leq 2\pi$

The graph below shows $f(x) = x^3 + 2x^2 - 4$ and $g(x) = 3x$, and the line $x = 1$. Find the area shown.



In a Sixth Form the probability that a chemistry student passes an assessed practical is 0.75. Out of a group of 30 students 12 passed. Should the head of science be concerned?

The decay in temperature of a cup of tea is modelled by a function of the form $Ae^{-0.02t}$. Given that the initial temperature of the tea (after adding milk) is $83°$ C, what is the value of A?

For $X \sim \mathrm{N}(120, 3^2)$ find
a) $\mathrm{P}(X = 124)$
b) $\mathrm{P}(X \leq 118)$
c) $\mathrm{P}(121 \leq X \leq 126)$
d) a such that $\mathrm{P}(X \leq a) = 0.4$

Find
$$\int 2x\sqrt{4x^2 - 16}\ \mathrm{d}x$$

What are the conditions for the binomial distribution to be a suitable model for a situation?

A player hits a high score in a computer game with a probability 0.35. Suggest a distribution for the number of times they hit a high score in 12 games.

Solve $4^{2+x} = 5^{x-1}$ giving your answer in terms of logarithms.

AQA A-Level Mathematics Warmup - Paper 3 2023 Answers

$$\frac{9}{15}$$

Using the quotient rule with $\tan(x) = \frac{\sin(x)}{\cos(x)}$.

$$\frac{dy}{dx} = \frac{\cos(x)\cos(x) + \sin(x)\sin(x)}{\cos^2(x)}$$
$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$
$$= \frac{1}{\cos^2(x)}$$
$$= \sec^2(x)$$

Always comments about dispersion and measures of location in questions like this. The median for females is lower than for males suggesting that females tend to buy cars with smaller engines. The spread (interquartile range) is larger for males than females.

a) $f(2) = -1.2274$ and $f(3) = 5.8875$. Since there is a sign change and $f(x)$ is continuous in this interval there is a root in $[2,3]$.

b) $x_{n+1} = x_n - \dfrac{x_n^2 \ln(x_n) - 4}{x_n + 2x_n \ln(x_n)}$

Using this iteration gives $x \approx 2.23202$

a) $\dfrac{21}{70}$

b) $\dfrac{1}{2}$

$\bar{x} = 4.4$

$\sigma = 2.017$

$\sum x = 154$

$\sum x^2 = 820$

$Q_1 = 2$

median $= 4$

$Q_3 = 6$

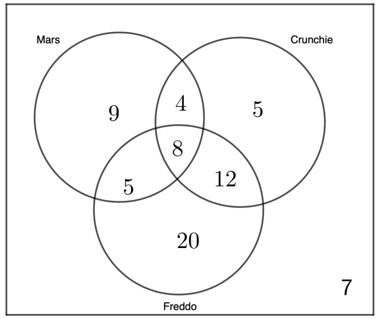
$5 - 2\sqrt{6}$ and $5 + 2\sqrt{6}$

Use the quotient rule with $u = 3x^3 + 2x$ and $v = \sqrt{2x+1}$ to get

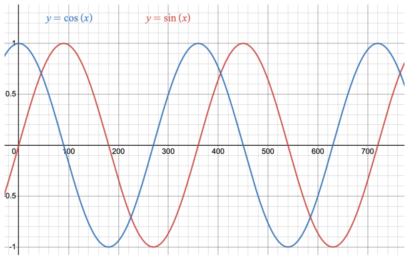
$$\frac{dy}{dx} = \frac{15x^3 + 9x^2 + 2x + 2}{(2x+1)^{\frac{3}{2}}}$$

a) 0.0834

b) $1 - P(X \leq 8) = 0.0153$



Divide the frequency by the width of the class interval.



$$\frac{-20}{3}$$

Let X be the number of people passing, then $X \sim B(30, 0.75)$. Let $H_0 : p = 0.75$, $H_1 : p < 0.75$. Then $P(X \leq 12) = 5.008 \times 10^{-5}$. $5.008 \times 10^{-5} < 0.05$ so at the $5\,\%$ level there is evidence to reject the null hypothesis in favour of the alternative. The head of science should be concerned.

$$83$$

a) 0

b) 0.2525

c) $0.9772 - 0.6305 = 0.3467$

d) 119.24

$$\int 2x\sqrt{4x^2 - 16}\,dx = \int 4x\sqrt{x^2 - 4}\,dx$$
$$= \frac{4}{3}(x^2 - 4)^{\frac{3}{2}} + C$$

- There are a fixed number, n, of trials.
- Each trial is independent.
- Two possible outcomes to each trial - success or failure
- Fixed probably of success

$X \sim B(12, 0.35)$ where X is the discrete random variable "number of games hitting high score"

$$x = \frac{-\left(4\log(2) + \log(5)\right)}{2\log(2) - \log(5)}$$