A - Level Maths 15 Minute Boost 3

What symbol is used to represent the rational numbers?	$\mathbb{Q}$
What is the discriminant for the quadratic $ax^2 + bx + c$?	$b^2 - 4ac$
$\sqrt{ab} =$	$\sqrt{a}\sqrt{b}$
How do you find the distance between $A(x_A, y_A)$ and $B(x_B, y_B)$?	$\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$
When is a sequence x_n decreasing?	$x_{n+1} < x_n \quad \forall n$

1) Find the cartesian form of the equation with parametric form $x = 2 + 3\cos(\theta)$ and $y = 1 + \sin(\theta)$

$x = 2 + 3\cos\theta \Rightarrow \frac{x-2}{3} = \cos(\theta)$

$y = 1 + \sin\theta \Rightarrow y - 1 = \sin(\theta)$

So $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{x-2}{3}\right)^2 + (y-1)^2 = 1$

$\Rightarrow (x-2)^2 + 9(y-1)^2 = 9$

2 a) Find $\int x^{-2}\ln(x)\,dx$

$\int u\,dv = uv - \int v\,du$

Let $u = \ln(x)$ $\quad dv = x^{-2}$

$\frac{du}{dx} = \frac{1}{x}$ $\quad v = -\frac{1}{x}$

So

$$\int x^{-2}\ln(x) = -\frac{\ln(x)}{x} - \int -\frac{1}{x}\frac{1}{x}\,dx$$

$$= -\frac{\ln(x)}{x} + \int \frac{1}{x^2}\,dx$$

$$= -\frac{\ln(x)}{x} - \frac{1}{x}$$

b) Hence, find the area of the region bounded by the function $f(x) = x^{-2}\ln(x)$, the $x-$axis and the lines $x = 1$ and $x = 3$.

$$\int_1^3 \frac{\ln(x)}{x^2}\,dx = \left[-\frac{\ln(x)}{x} - \frac{1}{x}\right]_1^3$$

$$= \left(-\frac{\ln(3)}{3} - \frac{1}{3}\right) - \left(\frac{-\ln(1)}{1} - 1\right)$$

$$= \frac{2}{3} - \frac{\ln(3)}{3}$$