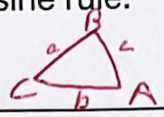
A - Level Maths 15 Minute Boost 1

State the cosine rule: 	$a^2 = b^2 + c^2 - 2bc\cos(A)$
What are the small angle approximations?	$\sin(x) \approx x$ $\cos(x) \approx 1 - \frac{x^2}{2}$ $\tan(x) \approx x$
$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin(ax+b)\right) =$	$a\cos(ax+b)$
What are the three Pythagorean trigonometric identities?	$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $\cot^2 x + 1 = \operatorname{cosec}^2 x$
$\int e^{ax+b}\,\mathrm{d}x =$	$\frac{e^{ax+b}}{a}$

1) Given that $(x+4)$ is a factor of $p(x) = x^3 + bx^2 - 2x - 24$ find the value of b and fully factorise $p(x)$.

$p(-4) = 0 \Rightarrow -64 + 16b + 8 - 24 = 0$

$16b = 8$

$b = 5$

$x^3 + 5x^2 - 2x - 24 = (x+4)(x^2 + x - 6)$

$= (x+4)(x-2)(x+3)$

2 a) Find the binomial expansion of $(4 + 3x)^{\frac{1}{2}}$ up to there term including x^3.

$$(4+3x)^{1/2} = 4^{1/2}\left(1 + \frac{3}{4}x\right)^{1/2}$$

$$= 2\left[1 + \frac{1}{2}\left(\frac{3x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}\left(\frac{3x}{4}\right)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}\left(\frac{3x}{4}\right)^3 + \cdots\right]$$

$$= 2 + \frac{3x}{4} - \frac{9x^2}{64} + \frac{27x^3}{512}$$

b) Using part (a), find an approximate value to $\int_0^1 (4 + 3x)^{\frac{1}{2}}\,dx$ and the percentage error made in this calculation. (Use your calculator to obtain the "exact" value of the. integral.)

$$\int_0^1 (4+3x)^{1/2}\,dx \approx \int_0^1 2 + \frac{3x}{4} - \frac{9x^2}{64} + \frac{27x^3}{512}\,dx$$

$$= \left[2x + \frac{3x^2}{8} - \frac{9x^2}{192} + \frac{27x^4}{2048}\right]_0^1$$

$$= 2.34131$$

$$\% \text{ error} = \left(\frac{2.34131 - 2.33783533}{2.33783533}\right) \times 100 \approx 0.16\%$$