A - Level Further Maths 15 Minute Boost 4

$\sum_{r=1}^{n} r^3 =$	$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$
For non-singular matrices $\mathbf{A}$ and $\mathbf{B}$ what is $(\mathbf{AB})^{-1} =$?	$(AB)^{-1} = B^{-1}A^{-1}$
Describe the loci $\lvert z - z_1 \rvert < r$ for complex numbers z and z_1	Locus of points within a circle centre z_1, radius r
$\sinh(2x) =$	$2\sinh(x)\cosh(x)$
What is the auxiliary equation for the 2nd order DE $2y'' + 6y' + 4y = 0$?	$\lambda^2 + 3\lambda + 2 = 0$

1 Find the equation of the plane containing the points $A(1,1,1)$, $B(3,1,2)$ and $C(1,4,1)$.

$\vec{AB} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ $\quad \vec{AC} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$

So $\underline{r} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$ Vector form of plane

or $\underline{n} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 0 & 1 \\ 0 & 3 & 0 \end{vmatrix} = \underline{i}(-3) - \underline{j}(0) + \underline{k}(6)$

$= \begin{pmatrix} -3 \\ 0 \\ 6 \end{pmatrix}$

Hence, $\underline{a} \cdot \underline{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 0 \\ 6 \end{pmatrix} = 1$, so

$\underline{r} \cdot \begin{pmatrix} -3 \\ 0 \\ 6 \end{pmatrix} = 1$ is the scalar product form of the plane.

Cartesian form:

$-x + 2z = 1$

2 Find the eigenvalue - eigenvector pairs for the matrix $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}$

$$\begin{vmatrix} 4-\lambda & 1 \\ -2 & 1-\lambda \end{vmatrix} = (4-\lambda)(1-\lambda) + 2$$
$$= \lambda^2 - 5\lambda + 6$$

For characteristic equation, $\det(A-\lambda I) = 0$, so

$$\lambda^2 - 5\lambda + 6 = 0$$
$$\Rightarrow (\lambda - 3)(\lambda + 2) = 0$$

So $\lambda_1 = 3$, $\lambda_2 = 2$

<u>When $\lambda = 3$</u>

$$\begin{pmatrix} 1 & 1 \\ -2 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underset{\sim}{0} \Rightarrow \begin{matrix} x_1 + x_2 = 0 \\ -2x_1 - 2x_2 = 0 \end{matrix}$$

Let $\underset{\sim}{v_1} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

<u>When $\lambda = 2$</u>

$$\begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underset{\sim}{0} \Rightarrow \begin{matrix} 2x_1 + x_2 = 0 \\ -2x_1 - x_2 = 0 \end{matrix}$$

So let $\underset{\sim}{v_2} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Then the eigenvalue-eigenvector pairs are

$$\lambda_1 = 3, \underset{\sim}{v_1} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$\lambda_2 = 2, \underset{\sim}{v_2} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$