A - Level Further Maths 15 Minute Boost 3

Define $\tanh(x)$ in terms of exponentials.	$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$
State the modulus-argument form for a complex number.	$z = r(\cos\theta + i\sin\theta)$
What is the 3 by 3 identity matrix?	$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
Find the scalar product of $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -4 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -4 \\ -1 \end{pmatrix} = 4 - 4 - 2 = -2$
$\sum_{r=1}^{n} r =$	$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

1 Find the matrix representing a rotation, centre the origin of $270°$ counter clockwise, followed by a reflection in the line $y = -x$.

Rotation $270°$ counter clockwise is $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Reflection in line $y = -x$ is $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

So combined transformation represented by

$$M = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

2 Prove by induction $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$

Step 1: When $n=1$ $\quad$ LHS $= 1^2 = 1$

RHS $= \frac{1}{6} \times 1 \times 2 \times 3 = 1$

Step 2: We assume true for $n=k$, ie

$$\sum_{r=1}^{k} r^2 = \frac{1}{6}k(k+1)(2k+1)$$

RTS: $\sum_{r=1}^{k+1} r^2 = \frac{1}{6}(k+1)(k+2)(2k+3)$

Step 3: Consider $n=k+1$

$$\sum_{r=1}^{k+1} r^2 = \sum_{r=1}^{k} r^2 + (k+1)^2$$

$$= \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$$ by inductive hypothesis

$$= \frac{1}{6}(k+1)\left[k(2k+1) + 6(k+1)\right]$$

$$= \frac{1}{6}(k+1)(2k^2 + k + 6k + 6)$$

$$= \frac{1}{6}(k+1)(2k+3)(k+2)$$

Step 4: We have shown the result to be true for $n=1$, and if true for $n=k$ then it is also true for $n=k+1$. Hence, by the principle of mathematical induction it is ~~also~~ true for all $n \in \mathbb{N}$

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