A - Level Further Maths 15 Minute Boost 2

Define $\tanh(x)$ in terms of exponential functions.	$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
How do you calculate the volume of revolution for a function rotated about the $x-$axis ?	$\text{Volume} = \pi \int_a^b y^2 \, dx$
What are the equations of the asymptotes for the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{x}{a} = \pm \frac{y}{b}$ $\Rightarrow$ $y = \pm \frac{b}{a} x$
Find $(1+i)^8$	$(1+i)^8$
Calculate $\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$	$\begin{vmatrix} i & j & k \\ 2 & 4 & 1 \\ 1 & 1 & 3 \end{vmatrix} = i(10) - j(5) + k(-2) = \begin{pmatrix} 10 \\ -5 \\ -2 \end{pmatrix}$

1 Find the vector equation of the line joining the points $(2,4,1)$ and $(5,-2,4)$.

Direction vector $= \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}$

Hence, eqn of line is

$\underset{\sim}{r} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}$

2 By finding the inverse of a matrix find the solution of the simultaneous equations given below.

$$\begin{cases} 3x + y + 4z = 25 \\ -2x + 4y + 3z = 20 \\ x + 5y - 2z = 9 \end{cases}$$

Matrix form is $\begin{pmatrix} 3 & 1 & 4 \\ -2 & 4 & 3 \\ 1 & 5 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 25 \\ 20 \\ 9 \end{pmatrix}$

Hence, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 & 1 & 4 \\ -2 & 4 & 3 \\ 1 & 5 & -2 \end{pmatrix}^{-1} \times \begin{pmatrix} 25 \\ 20 \\ 9 \end{pmatrix}$

$$= \frac{1}{126}\begin{pmatrix} 23 & -22 & 13 \\ 1 & 10 & 17 \\ 14 & 14 & -14 \end{pmatrix}\begin{pmatrix} 25 \\ 20 \\ 9 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$