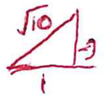
A - Level Further Maths 15 Minute Boost 1

Define $\cosh(x)$ in terms of exponential functions.	$\cosh(x) = \frac{e^{x}+e^{-x}}{2}$
How do you define the mean value of a function over the interval $[a, b]$.	$M := \frac{1}{b-a}\int_a^b f(x)\,dx$
What is a suitable concluding statement for a proof by induction?	We have shown the statement is true for the base case, and if true for $n=k$ then also true for $n=k+1$. Hence, by the principle of mathematical induction it is true $\forall n \in \mathbb{N}$ (or suitable)
State Viète's formulae for the cubic equation $ax^3 + bx^2 + cx + d = 0$	$\alpha+\beta+\gamma = -\frac{b}{a}$ $\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a}$ $\alpha\beta\gamma = -\frac{d}{a}$ where $\alpha, \beta,$ and γ are the roots.
What is the derivative of $y = \text{arsinh}(4x)$	$\frac{dy}{dx} = \frac{1}{\sqrt{1+(4x)^2}} \times 4 = \frac{4}{\sqrt{1+16x^2}}$

1 Find the Maclaurin series for the function $y = \ln(e + 3ex)$. You may use results given in the formula book.

$$y = \ln(e(1+3x))$$
$$= \ln(e) + \ln(1+3x)$$
$$= 1 + \left(3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3}\right)$$
$$= 1 + 3x - \frac{9x^2}{2} + 9x^3$$



$\tan\theta = 3 \Rightarrow 1 + 9 = \sec^2\theta \Rightarrow \cos^2\theta = \frac{1}{10}$

$\sin\theta =$

Hence $\cos 2\theta = 2\cos^2\theta - 1 = \frac{1}{5} - 1 = -\frac{4}{5}$

So $\sin(2\theta) = \frac{3}{5}$

2 a) What is the matrix representing a reflection in the line $y = 3x$?

$\tan\theta = 3 \Rightarrow \theta =$ eugh! see above!

$$M = \begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix}$$

b) Find all invariant lines of the form $y = mx + c$ for the matrix found in **(b)**?

Let $y = mx + c$, then

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix}\begin{pmatrix} x \\ mx + c \end{pmatrix} = \begin{pmatrix} -\frac{4}{5}x + \frac{3}{5}(mx + c) \\ \frac{3}{5}x + \frac{4}{5}(mx + c) \end{pmatrix}$$

$$\frac{3}{5}x + \frac{4}{5}(mx + c) = m\left(-\frac{4}{5}x + \frac{3}{5}(mx + c)\right) + c$$

$$\frac{3}{5}x + \frac{4}{5}mx + \frac{4}{5}c = -\frac{4}{5}mx + \frac{3}{5}m^2x + \frac{3}{5}mc + c$$

$$3x + 4mx + 4c = -4mx + 3m^2x + 3mc + 5c$$

$$0 = (3m^2 - 8m - 3)x + c(3m + 1)$$

$$\Rightarrow \quad 0 = (3m + 1)(m - 3) + c(3m + 1)$$

Hence

	m	c
$m = -\frac{1}{3}$	$-1/3$	anything
$m = 3$	3	0

So $y = 3x$ and $y = -\frac{1}{3}x + c$