

Edexcel FP1 2015

1. $f(x) = 9x^3 - 33x^2 - 55x - 25$

Given that $x=5$ is a solution of the equation $f(x)=0$, use an algebraic method to solve $f(x)=0$ completely.

(5)

2. In the interval $13 < x < 14$, the equation

$$3 + x \sin\left(\frac{x}{4}\right) = 0, \text{ where } x \text{ is measured in radians,}$$

has exactly one root, α .

(a) Starting with the interval $[13, 14]$, use interval bisection twice to find an interval of width 0.25 which contains α .

(3)

(b) Use linear interpolation once on the interval $[13, 14]$ to find an approximate value for α . Give your answer to 3 decimal places.

(4)

3. (a) Using the formulae for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$, show that

$$\sum_{r=1}^n (r+1)(r+4) = \frac{n}{3}(n+4)(n+5)$$

for all positive integers n .

(5)

(b) Hence show that

$$\sum_{r=n+1}^{2n} (r+1)(r+4) = \frac{n}{3}(n+1)(an+b)$$

where a and b are integers to be found.

(3)

4.

$$z_1 = 3i \text{ and } z_2 = \frac{6}{1+i\sqrt{3}}$$

(a) Express z_2 in the form $a + ib$, where a and b are real numbers.

(2)

(b) Find the modulus and the argument of z_2 , giving the argument in radians in terms of π .

(4)

(c) Show the three points representing z_1 , z_2 and $(z_1 + z_2)$ respectively, on a single Argand diagram.

(2)

5. The rectangular hyperbola H has equation $xy = 9$

The point A on H has coordinates $\left(6, \frac{3}{2}\right)$.

- (a) Show that the normal to H at the point A has equation

$$2y - 8x + 45 = 0 \quad (5)$$

The normal at A meets H again at the point B .

- (b) Find the coordinates of B .

(i) Find the modulus and the argument of z_1 , giving the argument in radians in terms of π . (4)

6. (i) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^n - 1) & 5^n \end{pmatrix} \quad (6)$$

- (ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^n (2r-1)^2 = \frac{1}{3}n(4n^2-1) \quad (6)$$

7. (i) $\mathbf{A} = \begin{pmatrix} 5k & 3k-1 \\ -3 & k+1 \end{pmatrix}$, where k is a real constant.

Given that \mathbf{A} is a singular matrix, find the possible values of k .

(4)

- (ii) $\mathbf{B} = \begin{pmatrix} 10 & 5 \\ -3 & 3 \end{pmatrix}$

A triangle T is transformed onto a triangle T' by the transformation represented by the matrix \mathbf{B} .

The vertices of triangle T' have coordinates $(0, 0)$, $(-20, 6)$ and $(10c, 6c)$, where c is a positive constant.

The area of triangle T' is 135 square units.

- (a) Find the matrix \mathbf{B}^{-1} (2)

- (b) Find the coordinates of the vertices of the triangle T , in terms of c where necessary. (3)

- (c) Find the value of c . (3)

8. The point $P(3p^2, 6p)$ lies on the parabola with equation $y^2 = 12x$ and the point S is the focus of this parabola.

(a) Prove that $SP = 3(1 + p^2)$ (3)

The point $Q(3q^2, 6q)$, $p \neq q$, also lies on this parabola.

The tangent to the parabola at the point P and the tangent to the parabola at the point Q meet at the point R .

(b) Find the equations of these two tangents and hence find the coordinates of the point R , giving the coordinates in their simplest form. (8)

(c) Prove that $SR^2 = SP \cdot SQ$ (3)