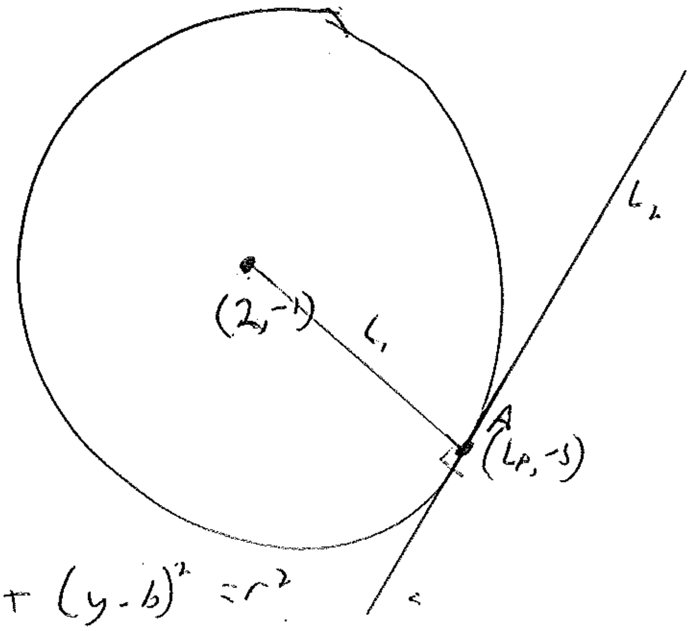
1) $\left(2 - \frac{x}{4}\right)^{10} = {}^{10}C_0\, 2^{10}\left(-\frac{x}{4}\right)^0 + {}^{10}C_1\, 2^9\left(-\frac{x}{4}\right) + {}^{10}C_2\, 2^8\left(-\frac{x}{4}\right)^2$

$= 1024 - 10 \times 512 \times -\frac{x}{4} + 45 \times 256 \times \left(-\frac{x}{4}\right)^2$

$= 1024 - 1280x + 720x^2$

2)



a) $(x-a)^2 + (y-b)^2 = r^2$

$r = \sqrt{2^2 + 4^2} = \sqrt{20}$

so $(x-2)^2 + (y+1)^2 = 20$

b) Tangents to the circle are perpendicular to radii

Gradient of $L_1 = \frac{-4}{+2} = -2$

$\therefore$ gradient of tangent is $\frac{1}{2}$

so $y = mx + c$

with $m = \frac{1}{2}$ passing through $(4,-5)$

$-5 = 4 \times \frac{1}{2} + c \quad \Rightarrow c = -7$

$y = \frac{1}{2}x - 7$

$\Rightarrow x - 2y - 14 = 0$

3) a) $f(-1) = 6(-1)^3 + 3(-1)^2 - A + B$

$= -6 + 3 - A + B$

$= -3 - A + B$

As $f(-1) = 45$

$-3 - A + B = 45$

$\Rightarrow B - A = 48$ ①

b) If $(2x+1)$ is a factor of $f(x)$ then $f(-\frac{1}{2}) = 0$, so

$6\left(\frac{-1}{2}\right)^3 + 3\left(\frac{-1}{2}\right)^2 - \frac{A}{2} + B = 0$

$\frac{-3}{4} + \frac{3}{4} - \frac{A}{2} + B = 0$

$\Rightarrow 2B - A = 0 \Rightarrow 2B = A$ ②

② into ①

$B - 2B = 48 \Rightarrow B = -48$

Hence $A = -96$

$\therefore f(x) = 6x^3 + 3x^2 - 96x - 48$

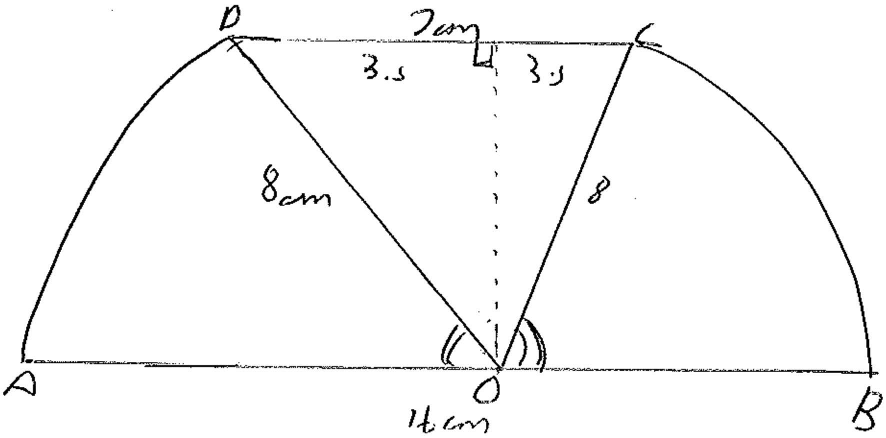
$= (2x+1)(3x^2 + 0x - 48)$

$= \cancel{(2x+1)(3x}$

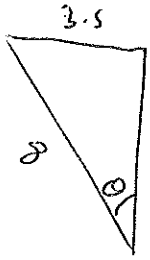
$= (2x+1)3(x^2 - 16)$

$= 3(2x+1)(x-4)(x+4)$

4)



a)



$\sin\theta = \frac{3.5}{8} \Rightarrow \theta \approx 0.4528165967$

$\Rightarrow \angle COD = 2\theta$

$= 0.9056331895$

$= 0.906$

b) $\angle AOD = \angle BOC \doteq (\pi - 0.906)/2$

≈ 1.12

Perimeter $= 7 + 16 + 2\times(8\times 1.12)$

$= 40.88767371$

$\approx 40.9\text{cm}$

d) Area = 2x sector + area of triangle

$= 2\times(8^2\times 1.12)\times\frac{1}{2} + \frac{1}{2}\times 8\times 8\times \sin(0.906)$

$= 96.72881835$

$= 96.7\text{cm}^2$

5) i) Let a = first term
r = common ratio

Then
$U_1 = a$
$U_2 = ar$

so $a + ar = 34 \Rightarrow a(1+r) = 34 \Rightarrow a = \frac{34}{1+r}$ ①

Sum to infinity, $S_\infty = 162 \Rightarrow \frac{a}{1-r} = 162$ ②

① into ②

$$\frac{34}{(1-r)(1+r)} = 162$$

$$\frac{34}{1-r^2} = 162$$

$$\Rightarrow 34 = 162 - 162r^2$$

$$162r^2 = 128$$

$$r^2 = \frac{64}{81}$$

so $r = \frac{8}{9}$

Hence

$$a = \frac{34}{\frac{17}{9}} = \frac{9 \times 34}{17} = 18$$

So $a = 18$, $r = \frac{8}{9}$

ii) $a = 42$, $r = \frac{6}{7}$ $S_n \geq 290$ since we want smallest n such that S_n exceeds 290

$$S_n = \frac{a(1-r^n)}{1-r}$$

so $290 = \frac{42(1-(\frac{6}{7})^n)}{1/7} \Rightarrow 2030 = 42(1-(\frac{6}{7})^n)$ STUPID!!

$\frac{145}{3} = 1 - (\frac{6}{7})^n$

s

ii) $a = 42$, $r = \frac{6}{7}$

$$S_n = \frac{a(1-r^n)}{1-r}$$ with $S_n = 290$ since we want the ~~biggest~~ smallest n s.t. $S_n \geq 290$

$$290 = \frac{42\left(1-\left(\frac{6}{7}\right)^n\right)}{\frac{1}{7}}$$

$$\Rightarrow \frac{290}{294} = \left(1-\left(\frac{6}{7}\right)^n\right)$$

so $$\left(\frac{6}{7}\right)^n = \frac{2}{147}$$

$$n \log_{10}\left(\frac{6}{7}\right) = \log_{10}\left(\frac{2}{147}\right)$$

hence
$$n = \frac{\log_{10}\left(\frac{2}{147}\right)}{\log_{10}\left(\frac{6}{7}\right)}$$

$$= 27.87717453$$

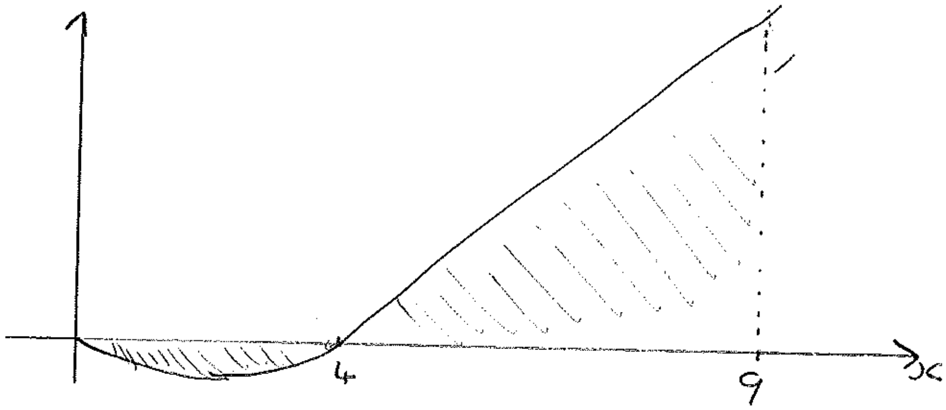
So the least value of n when S_n exceeds 290 is 28.

6) a) $\int 10x(x^{1/2} - 2)\,dx = \int 10x^{3/2} - 20x \, dx$

$$= \frac{10}{5/2} x^{5/2} - \frac{20x^2}{2} + c$$

$$= 4x^{5/2} - 10x^2 + c$$

b)



$$\text{Total area} = \underbrace{\left| \int_0^4 10x(x^{1/2} - 2)\,dx \right|}_{A} + \underbrace{\int_4^9 10x(x^{1/2} - 2)\,dx}_{B}$$

$$A = \left[4x^{5/2} - 10x^2\right]_0^4 = -32$$

$$B = \left[4x^{5/2} - 10x^2\right]_4^9 = 162 - -32$$

$$= 194$$

$\therefore$ Total area $= |-32| + 194$

$= 226$

7) i) $8^{2x+1} = 24$

$\Rightarrow (2x+1)\log_{10} 8 = \log_{10} 24$

$2x+1 = \dfrac{\log_{10} 24}{\log_{10} 8}$

$x = \dfrac{\dfrac{\log_{10} 24}{\log_{10} 8} - 1}{2}$

$= 0.264160416 8$

$= 0.264$

ii) $\log_2 (11y - 3) - \log_2 3 - 2\log_2 y = 1$ $\quad , y > \frac{3}{11}$

$\log_2 (11y - 3) - \log_2 3 - \log_2 y^2 = 1$

$\log_2 (11y - 3) - (\log_2 3 + \log_2 y^2) = 1$

$\log_2 (11y - 3) - \log_2 (3y^2) = 1$

$\log_2 \left(\dfrac{11y - 3}{3y^2}\right) = 1$

$\dfrac{11y - 3}{3y^2} = 2$

$11y - 3 = 6y^2$

$\Rightarrow 6y^2 - 11y + 3 = 0$

$(2y - 3)(3y - 1) = 0$

$\Rightarrow y = \frac{3}{2}$ and $y = \frac{1}{3}$

8) i) $\sin 3\theta - \sqrt{3}\cos 3\theta = 0$

$\Rightarrow \sin 3\theta = \sqrt{3}\cos 3\theta$

so $\tan 3\theta = \sqrt{3}$

$3\theta = \frac{\pi}{3}$

Hence $\theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$ since the period of $\tan\theta$ is π.

ii) a) $4\sin^2 x + \cos x = 4 - k$

$(\sin^2 x + \cos^2 x = 1)$

$4(1 - \cos^2 x) + \cos x = 4 - k$

$4 - 4\cos^3 x + \cos x = 4 - k$

$4\cos^2 x - \cos x - k = 0$

Let $c = \cos^2 x$ then $4c^2 - c - k = 0$

$c = \frac{1 \pm \sqrt{1^2 - 4 \cdot 4 \cdot -k}}{8}$

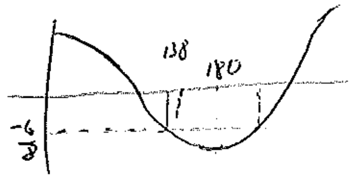
$= \frac{1 \pm \sqrt{1 + 16k}}{8}$

b) When $k = 3$

$\cos x = \frac{1 \pm \sqrt{49}}{8} = \frac{1 \pm 7}{8}$ so, $\cos x = 1$ or $\cos x = \frac{-6}{8}$

$\cos x = 1 \Rightarrow x = 0$

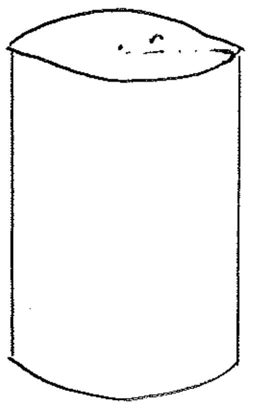
$\cos x = \frac{-6}{8} \Rightarrow x = 138.59°$



or $180 + (180 - 138.59)$
$= 221.41°$

so $x = 0°, 138.59°, 221.41°$

9)



a)

Volume $= \pi r^2 h$ ①

Surface area $= 2\pi r^2 + 2\pi r h$ ②

As volume $= 75\pi$ ① gives $h = \dfrac{75}{r^2}$

Then surface area $= 2\pi r^2 + 2\pi \cdot \dfrac{75}{r}$

$= 2\pi r^2 + \dfrac{150\pi}{r}$

And cost is

$C = 3 \times 2\pi r^2 + \dfrac{2 \times 150\pi}{r}$

$= 6\pi r^2 + \dfrac{300\pi}{r}$

b) $\dfrac{dC}{dr} = 12\pi r - \dfrac{300\pi}{r^2}$

At a minimum $\dfrac{dC}{dr} = 0$

$12\pi r - \dfrac{300\pi}{r^2} = 0$

$12\pi r = \dfrac{300\pi}{r^2}$

$$12\pi r^3 = 300\pi$$

$$r^3 = 25$$

so $r = \sqrt[3]{25}$

$= 2.924017738$

$\therefore$ minimum cost $= 6\pi(25)^{2/3} + \dfrac{300\pi}{25^{1/3}}$

$= 483.4843085$

$= \pounds 483$

ii) $\dfrac{d^2C}{dr^2} = 12\pi + \dfrac{600\pi}{r^3}$

at $r = 25^{1/3}$

$\dfrac{d^2C}{dr^2} = 36\pi$

$> 0 \Rightarrow$ minimum at $r = 25^{1/3}$