1)

a) $(2\sqrt{5})^2 = 4 \cdot 5$

$= 20$

b) $\dfrac{\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}} = \dfrac{\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}} \cdot \dfrac{2\sqrt{5} + 3\sqrt{2}}{2\sqrt{5} + 3\sqrt{2}}$

$= \dfrac{2\sqrt{5}\sqrt{2} + 6}{20 - 18}$

$= \dfrac{2\sqrt{5}\sqrt{2} + 6}{2}$

$= 3 + \sqrt{2}\sqrt{5}$

$= 3 + \sqrt{10}$

2) $y - 2x - 4 = 0$ ①

$4x^2 + y^2 + 20x = 0$ ②

① $\Rightarrow$ $y = 2x + 4$ - substitute this into ②

$4x^2 + (2x+4)^2 + 20x = 0$

$4x^2 + 4x^2 + 16x + 16 + 20x = 0$

$8x^2 + 36x + 16 = 0$

$4(2x^2 + 9x + 4) = 0$

$4(2x+1)(x+4) = 0$ $\Rightarrow x = \frac{-1}{2}$ or $x = {}^{-}4$

When $x = \frac{-1}{2}$ $y = 2 . \frac{-1}{2} + 4 = 3$

When $x = {}^{-}4$ $y = 2 . {}^{-}4 + 4 = {}^{-}4$

So the solution pairs are $\left(\frac{-1}{2}, 3\right)$

and $({}^{-}4, {}^{-}4)$

3) a) $y = 4x^3 - \frac{5}{x^2}$

$= 4x^3 - 5x^{-2}$

$\frac{dy}{dx} = 12x^2 + 10x^{-3}$

$= 12x^2 + \frac{10}{x^3}$

b) $\int y \, dx = \int 4x^3 - \frac{5}{x^2} \, dx$

$= x^4 + \frac{5}{x} + c$

4) i)

a) $U_3 = 2U_2 - U_1$

$= 2.4 - 4$

$= 4$

b) $$\sum_{n=1}^{20} U_n = 20 \times 4$$

$$= 80$$

ii)

a) $V_3 = 2V_2 - V_1$

$= 4k - k$

$= 3k$

$V_4 = 2V_3 - V_2$

$= 6k - 2k$

$= 4k$

b) $$\sum_{n=1}^{5} V_n = V_1 + V_2 + V_3 + V_4 + V_5 = 165$$

so

$k + 2k + 3k + 4k + 5k = 165$

$15k = 165$

$k = 11$

5) a) If no real roots then discriminant < 0

so with

$$(p-1)x^2 + 4x + (p-5) = 0$$

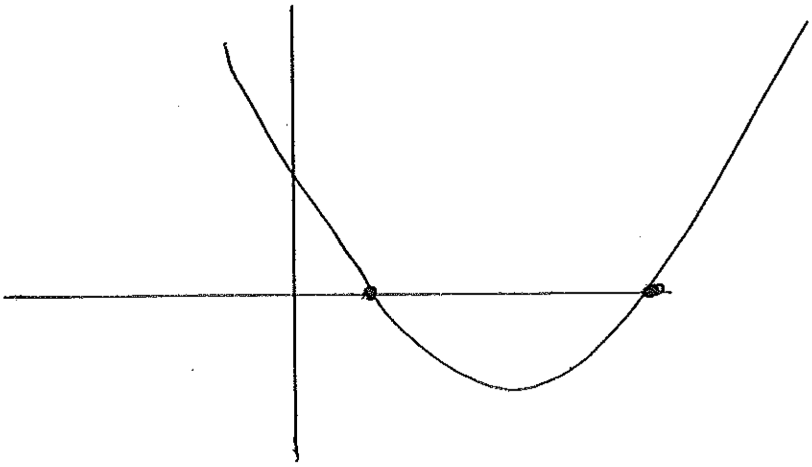
we have

$$b^2 - 4ac = 16 - 4(p-1)(p-5)$$
$$= 16 - 4(p^2 - 6p + 5)$$
$$= 16 - 4p^2 + 24p - 20$$
$$= -4p^2 + 24p - 4$$

so $\quad 4(-p^2 + 6p - 1) < 0$

$$\Rightarrow p^2 - 6p + 1 > 0$$

b)



$$p^2 - 6p + 1 = (p-3)^2 + 8$$

so $\quad p - 3 = 2\sqrt{2}$

$$p = 3 \pm 2\sqrt{2}$$

So crosses axis at $3 + 2\sqrt{2}$ and $3 - 2\sqrt{2}$

Hence

$$p < 3 - 2\sqrt{2}$$

and $\quad p > 3 + 2\sqrt{2}$

6) a) $y = \dfrac{(x^2+4)(x-3)}{2x}$

$= \dfrac{x^3 + 4x - 3x^2 - 12}{2x}$

$= \dfrac{x^2}{2} - \dfrac{3x}{2} + 2 - \dfrac{6}{x}$

$\dfrac{dy}{dx} = x - \dfrac{3}{2} + \dfrac{6}{x^2}$

b)

At $x = -1$ $\dfrac{dy}{dx} = -1 - \dfrac{3}{2} + 6$

$= \dfrac{7}{2}$

So the gradient of the tangent at $x = -1$ is $\dfrac{7}{2}$

when $x = -1$, $y = \dfrac{5 \times -4}{-2} = 10$

So with $y = \dfrac{7}{2}x + c$, substitute in $(\frac{7}{2}, 10)$

$10 = \dfrac{7}{2} \times -1 + c$

$\Rightarrow c = 10 + \dfrac{7}{2} = \dfrac{27}{2}$

so $y = \dfrac{7}{2}x + \dfrac{27}{2}$

$\Rightarrow 7x - 2y + 27 = 0$

7) a) $4^x = (2^2)^x$

$= 2^{2x}$

$= (2^x)^2$

$= y^2$

b) $8(4^x) - 9(2^x) + 1 = 0$

so $8y^2 - 9y + 1 = 0$

$\Rightarrow (8y - 1)(y - 1) = 0$

so $y = 1$ or $\frac{1}{8}$

When $y = 1$

$2^x = 1 \Rightarrow x = 0$

When $y = \frac{1}{8}$

$2^x = \frac{1}{8}$

ie

$2^x = \frac{1}{2^3}$

$= 2^{-3}$

$\Rightarrow x = -3$

so $x = -3$ and 0

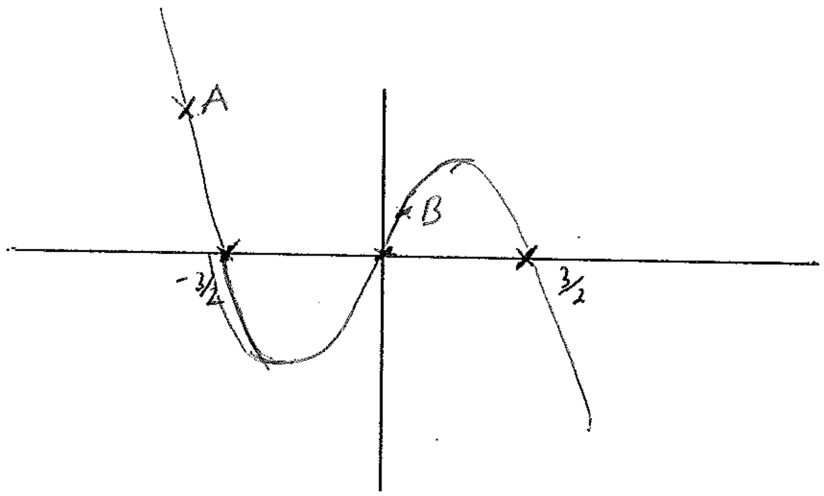
8 a) $9x - 4x^3 = x(9-4x^2)$

$= x(-2x+3)(2x+3)$

b) Crosses the axis at $x = 0$

$x = \frac{3}{2}$

$x = -\frac{3}{2}$



When $x = 1$ $y = 5$

$x = -1$ $y = -5$

c) At $x = -2$, $y = -18 + 32 = 14$, $A(-2, 14)$

$x = 1$, $y = 9 - 4 = 5$ $B(1, 5)$

Length AB

$= \sqrt{(-2-1)^2 + (14-5)^2}$

$= \sqrt{3^2 + 9^2}$

$= \sqrt{9+81}$

$= \sqrt{90}$

$= \sqrt{9 \times 10}$

$= 3\sqrt{10}$

8) a) Arithmetic series, first term a, common difference d.

$$u_n = a + (n-1)d$$

so

$$32000 = 17000 + (n-1) \times 1500$$

$$\frac{32000 - 17000}{1500} = n - 1$$

$$\Rightarrow n = 10 + 1$$
$$= 11$$

So $k = 11$

b) 11 years of arithmetic sequence

9 at 32000 a year

$$\cancel{S_{11} = \frac{11}{2}(32000}$$

$$S_{11} = \frac{11}{2}(2 \times 17000 + 10 . 1500)$$

$$= \frac{11}{2} \times 49000$$

$$= 269500$$

$$9 \times 32000 = 288000$$

So total earnings are

$$288000 + 269500 = £557500$$

10)

a) $f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2$

$= \frac{3}{2}x^{1/2} - \frac{9}{4}x^{-1/2} + 2$

$f(x) = \int f'(x)\,dx$

$= \frac{\frac{3}{2}x^{3/2}}{\frac{3}{2}} - \frac{\frac{9}{4}x^{1/2}}{\frac{1}{2}} + 2x + c$

$= x^{3/2} - \frac{9}{2}x^{1/2} + 2x + c$

$f(x)$ passes through $(4, 9)$

$9 = 4^{3/2} - \frac{9}{2} \cdot 4^{1/2} + 2 \cdot 4 + c$

$= 8 - 9 + 8 + c$

$= 7 + c$

$\Rightarrow c = 2$

so $f(x) = x^{3/2} - \frac{9}{2}x^{1/2} + 2x + 2$

b) $2y + x = 0 \quad \Rightarrow y = -\frac{1}{2}x$

So gradient of normal $= -\frac{1}{2}$

$\Rightarrow$ gradient of tangent $= 2$

Gradient of tangent is given by $f'(x)$

so

$2 = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2$

$\Rightarrow \frac{9}{4\sqrt{x}} = \frac{3\sqrt{x}}{2} \quad \Rightarrow 18 = 12x \quad \Rightarrow x = \frac{18}{12} = \frac{3}{2}$