Christmas Calculated Colouring - C1

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1 Introduction

Each question identifies a region or regions on the picture. Work out the answer and use the key to work out which colour to shade that region.

2 The Questions

1. $\int_0^2 2x + \frac{1}{2}\ \mathrm{d}x$.
2. The coefficient of the x term of $\frac{\mathrm{d}}{\mathrm{d}x}\left(7x^3 + 3x^2 + 5x\right)$.
3. Find x if $9^{2x} = 27^{x+1}$.
4. $64^{\frac{2}{3}}$.
5. The y intercept of the line connecting $(2, 4)$ and $(5, 1)$.
6. The gradient of the line $y = 5x + 3$
7. $\sqrt[5]{243}$
8. The b in the completed square form $(a(x+b)^2 + c)$ of the quadratic $3x^2 + 36x + 8$.
9. The discriminant of the quadratic $x^2 - 4x + 4$.
10. The third term of the sequence generated by the recurrence relation $x_{n+1} = x_n^2$ where the first term is $x_1 = 2$.
11. The greatest root of the equation $x^2 - 3x - 10 = 0$.
12. $27^{\frac{1}{3}}$.
13. The x coordinate of the turning point of $y = x^2 - 10x + 25$.
14. The x solution to the following pair of equations:

$$x + y = 9$$
$$3x + 4y = 33$$

.

15. The value you obtain when you evaluate $y = 3x^3 - 7x + 7$ at $x = 1$.

16. The y coordinate of the stationary point of $y = x^2 - 8x + 32$

17. Gradient of the tangent to the curve $y = x^3 + 2$ at $x = 1$.

18. Absolute value of the constant term in the equation of the tangent to the curve $y = x^3 + 4x^2 + 3$ at $x = 1$.

19. The sum of the 3rd and 4th triangular numbers.

20. The distance between the points $(21, 39)$ and $(24, 35)$.

21. The y solution to the following pair of equations:

$$x + y = 9$$
$$3x + 4y = 33$$

.

22. The gradient of the normal to the line $y = -\frac{1}{5}x + 10$ which passes through the point $(1, 8)$.

23. The y-intercept of the normal to the line $y = -\frac{1}{5}x + 10$ which passes through the point $(1, 8)$.

24. $\sqrt[6]{46656}$.

25. The coefficient of the x^2 term in $(x + 1)^3$.

26. $\frac{1}{4^{-2}}$.

27. The a when you write $\sqrt{63}$ in the form $a\sqrt{b}$.

28. The y intercept of the straight line which intersects $y = x^2$ at $x = -2$ and $x = 2.5$.

29. The coefficient of the x^3 term when you evaluate $\int 18x^2 \, \mathrm{d}x$.

30. The y coordinate of the point of intersection of the line $y = -3x + 21$ with the line $y = \frac{6}{7}x + \frac{12}{7}$.

31. The x coordinate of the point of intersection of the line $y = -3x + 21$ with the line $y = \frac{6}{7}x + \frac{12}{7}$.

32. The b when you write $\sqrt{27}$ in the form $a\sqrt{b}$.

33. The a when you write $\sqrt{27}$ in the form $a\sqrt{b}$.

34. The gradient of the line joining the points $(1, 3)$ and $(3, 13)$.

35. The second order derivative of $y = \frac{1}{3}x^3 + \frac{9}{2}x^2 + 18x$ evaluated at $x = -3$.

36. The largest integer satisfying $8(2x - 4) - 9x \leq 3$.

37. The denominator when you have rationalised the denominator of the following expression: $\frac{1}{\sqrt{11}-4}$.

38. The third term of the sequence given by $u_n = 6n - 2$.

39. $\int_0^5 \frac{3}{25}x^2 \, \mathrm{d}x$.

40. The power of x when you simplify $3x^2y \times 6x^3y^5$.

41. The a when you write $\sqrt{50}$ in the form $a\sqrt{b}$.

42. The power of y when you simplify $3x^2y \times 6x^3y^5$.

43. Let $f(x) = x^2 - 8x + 20$. Find the y coordinate of the turning point of $f(x + 1)$.

44. The x solution of the following pair of simultaneous equations:

$$\begin{aligned} x + 3y &= 11 \\ 4x - 7y &= 6 \end{aligned}$$

.

45. The smallest x value that satisfies the following pair of simultaneous equations:

$$\begin{aligned} x + y &= 11 \\ xy &= 30 \end{aligned}$$

.

46. The constant appearing in the expanded form of $(x - 2)^4$.

47. The largest b value that solves the following pair of simultaneous equations:

$$\begin{aligned} 3a + b &= 8 \\ 3a^2 + b^2 &= 28 \end{aligned}$$

.

48. The largest a value that solves the following pair of simultaneous equations:

$$\begin{aligned} 3a + b &= 8 \\ 3a^2 + b^2 &= 28 \end{aligned}$$

.

49. The y-intercept of the line joining $(-1, 7)$ to $(\frac{1}{2}, 4)$.

50. For the function $f(x) = x^2 - 2x + 4$ what is the value a such that $f(x + a)$ has the turning point $(-2, 3)$.

51. Find the common difference of the arithmetic series with second term 6 and the sum of the first four terms being 34.

52. The positive root of $y = 2x^2 - 9x - 18$.

53. A third of the y-intercept of the parabola $y = x^2 - 6x + 9$.

54. The derivative of $y = \frac{3}{2}x^2 + 3x$ evaluated at $x = 4$.

55. The positive y coordinate for a point of intersection of the curve $y = x^2 + 1$ and the line $y = -\frac{1}{2}x + 6$.

56. The denominator (when expressed as an improper fraction in simplified form) of the area enclosed by the x-axis, the curve $x = y^2 + 4$ and the tangent to the curve at the point $T(8, 2)$.

57. The 4th term of an arithmetic sequence such that the second term is 6 and the sum of the first 5 terms is 55.

58. The value you obtain when you calculate $\int_2^4 2x^3 \, \mathrm{d}x$ then divide by 8.

59. $\sqrt[3]{125}$

60. Given that $t^{\frac{1}{3}} = y$ what is the coefficient of y^{-1} in the expression $6t^{-\frac{1}{3}}$

61. With reference to Figure 1 which shows a sketch of the curve $y = 2x - x^2$ meeting the line $y = -2x$ at the origin O and the point P. Determine the coordinates of P and so find the area of the shaded region. Divide your answer by $\frac{32}{9}$.

62. The least root of the quadratic $y = x^2 - 12x + 35$.

63. The smallest positive root of the cubic $y = x^3 - 4x^2 - 17x + 60$.

64. The value of the second term of the series defined by $a_1 = 2$, $a_{n+1} = 2a_n - 1$.

65. Curve C is a function of x (i.e. $y = f(x)$) such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^3 + 2x - 7$$

Determine the curve C given that the point $P(2, 4)$ lies on C. Hence find an equation for the normal at P in the form $ax + by + c = 0$, where a, b, c are integers. Your answer is the value of b

66. The straight line l_1 has equation $y = 3x - 6$. The straight line l_2 is such that it is perpendicular to l_1 and passes through the point $(6, 2)$. Given that l_1 crosses the x-axis at A, l_2 crosses the x-axis at B and that l_1 intersects l_2 at C. Find the area of the triangle ABC.

67. For the series with first term 9 and common difference 6, what is the value l such that the sum of the first l terms is 105.

68. The value a when you express $\sqrt{1575}$ in the form $a\sqrt{b}$.

69. Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x - 3x^2$, and $x \geq 1$, and also that $y = 6$ at $x = 1$, find the greatest possible value of y.

70. The simplified form of $\frac{\sqrt{20}}{2}$ squared.

71. The coefficient of the x^2 term when you expand $(3x - 5)x^2$.

72. $2 \times 256^{\frac{1}{4}}$

73. The value you obtain when you multiply the x coefficient and the x^2 coefficient when you work out $\frac{d}{dx}(4x^2 + \frac{2}{3}x^3)$.

74. The coefficient of the x^4 term when you expand $(1 + x)^6$.

75. Let $f(x) = x^2 + 4x + 6$. Find the value of a such that $af(x)$ has a turning point with y coordinate 8.

76. The b when you express $\sqrt{125}$ in the form $a\sqrt{b}$.

77. $\frac{1}{40}\sum_{i=1}^{15} 5i$.

78. The least positive root of $x^3 - 14x^2 + 63x - 90$.

79. $\int_0^4 \frac{x^3}{4}\ \mathrm{d}x$.

80. The product of the roots of the quadratic $x^2 - 16x + 15$.

81. The a When $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$ is expressed in the form $a\sqrt{b}$.

82. The gradient of the curve $y = 2x^4 + 2x^3 + x$ evaluated at $x = 1$.

83. The greatest value of v such that u and v solve the following pair of simultaneous equations:

$$\begin{aligned} 2u + 2v &= 16 \\ uv &= 15 \end{aligned}$$

84. If the sum of the first n terms of a series is n^2+3n, by finding an expression for the nth term (or otherwise) find the first term of the underlying series.

85. What is the value of a when you write $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ in the form $a + b\sqrt{c}$.

86. The value of b when you write $\sqrt{75} + 2\sqrt{48} - 5\sqrt{12}$ in the form $a\sqrt{b}$.

87. The most positive root of the cubic $x^3 - 14x^2 + 63x - 90$.

88. The sum of the roots of the quadratic $x^2 - 5x + 6$.

89. The gradient of the line passing through the points $(-2, -2)$ and $(-1, 2)$.

90. The common difference of the series $9, 14, 19, 24, \ldots$.

91. The first term of the series with common difference 6 and fourth term 33.

92. The product of the roots of the quadratic $x^2 - 10x + 16$.

93. The y coordinate of the common point of intersection of the line $y = -\frac{1}{2}x + 6$ and the parabolas $y = x^2 + 1$ and $y = x^2 - 8x + 17$.

94. The coefficient of the x^4 term when you differentiate $y = 3x^5 - \frac{4}{x^2}$.

95. The value of the smallest u such that u and v solve the following pair of simultaneous equations:

$$\begin{aligned} 2u + 2v &= 16 \\ uv &= 15 \end{aligned}$$

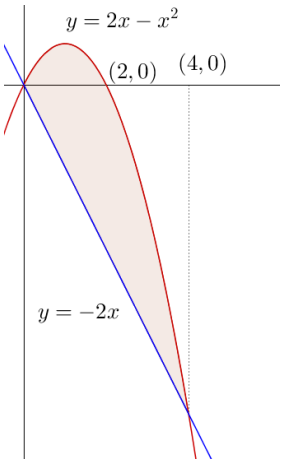

Figure 1: Picture for Question 61

96. The b in the completed square form $(a(x + b)^2 + c)$ of the quadratic $x^2 + 10x + 23$.

97. The power of z when you simplify $3x^2yz \times 2yz^3 \times x^2z^{-1}$.

98. $\frac{1}{2}\sqrt{256}$

99. The third term of the arithmetic series with second term 9 and the sum of the first 10 terms is 335.

100. The x coordinate of the point of intersection of the lines $l_1 : 7x + 2y = 25$ and $l_2 : 9x - 7y = 13$.