AQA Paper 2 2019 “Predicted Content”

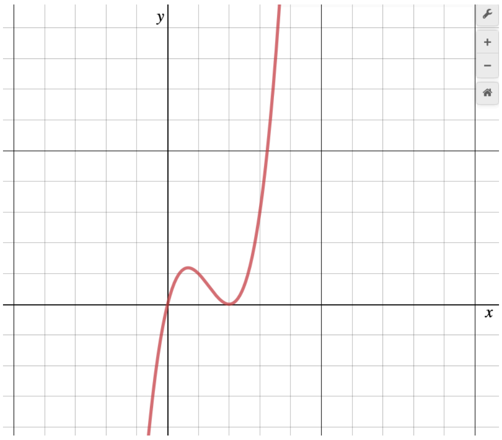
Name: ....

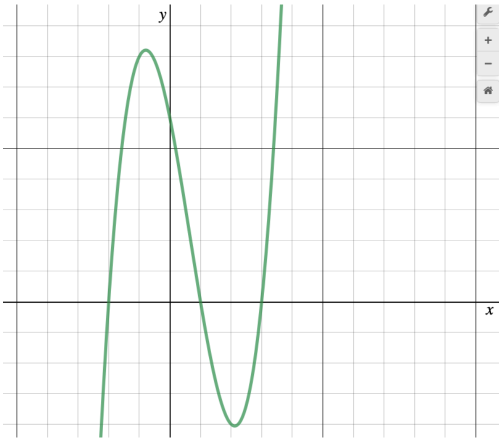
Class: ....

Mark: / 112

1 The function $f(x)$ is cubic with two positive roots and the coefficient of x^3 is positive. Which of the graphs below could be $f(x)$?

[1 mark]





2 The mass m of a radioactive substance decays following the equation $m = m_0 e^{-At}$. Justin wants to use a straight line to determine A. To do this he should plot,

[1 mark]

a) m against t

b) $\ln(m)$ against $\ln(t)$

c) $\ln(m)$ against t

d) m against $\ln(t)$

3 Using the quotient rule show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\cot(x)\right) = -\operatorname{cosec}^2(x)$

[4 marks]

4 Tom claims that for $n \geq 2$, $2^n - 1$ is prime,
Decide if this claim is true. If true provide a proof, or if you decide that it is not true explain how you know.

[3 marks]

5 **a)** Given that, for $f(x) = x^3 + 5x^2 + 8x + 4$, $f(-2) = 0$, factorise $f(x)$ fully.

[2 marks]

b) Hence find $\displaystyle\int \frac{5x^2 + 16x + 13}{x^3 + 5x^2 + 8x + 4}\, \mathrm{d}x$

[6 marks]

6 A graphic designer is modelling the profile of the roof of a circus tent with the function $y = \mathrm{e}^{\frac{x}{2}} + 1$.

a) Describe a sequence of transformations to obtain the graph of $f(x) = \mathrm{e}^{\frac{x}{2}} + 1$ from that of $g(x) = \mathrm{e}^{x}$.

[2 marks]

b) Sketch $f(x)$, showing any intersection points with the axes.

[2 marks]

c) Explain, fully justifying your answer, why $f(x)$ has no stationary point.

[3 marks]

d) The designer wishes to make a 3D model of this roof using foam. To do this he rotates the curve $y = e^{\frac{x}{2}} + 1$ about the $x-$axis. The volume of foam required is given by $\pi \int_0^4 (f(x))^2 \, dx$.

Use the trapezium rule with 4 strips to approximate the volume of foam required.

[5 marks]

e) Will the Trapezium rule produce an under estimate or over estimate of the true volume in this case. Explain your answer.

[2 marks]

7 A circle passes through the three points $A(6, -1)$, $B(4, -5)$ and $C(2,1)$.

a) Find the equation of the circle, clearly stating the centre and radius of the circle.

[7 marks]

b) The tangent to the circle at the point $D(4,1)$ crosses the $x-$axis at E and the $y-$axis at F. Find the area of the triangle OEF.

[5 marks]

8 Show that $\int_1^4 x\sqrt{3x+1}\ \mathrm{d}x = \frac{4}{135}\left(221\sqrt{13} - 18\right)$

[5 marks]

9 A curve has parametric equations $x = t^2 + 1$, $y = t^3 - 2t$.

a) Find the equation of the tangent at the point where $t = 1$.

[4 marks]

b) Find the coordinate of the point where this tangent meets the curve again.

[4 marks]

10 The number of bacteria in a petri dish is modelled mathematically by saying that the number of bacteria, n, increases at a rate proportional to the number of bacteria present at a given time. Suppose that the number of bacteria at 9 am is found to be 125. At 10 am there are 320 bacteria. At what time will there first be more than 2000 bacteria.

[6 marks]

11 Find the cartesian equation for the curve with parametric equations $x = \sin(\theta), y = 3\sin(2\theta)$.

[3 marks]

12 The vertical displacement of a particle attached to the end of a spring is given by the equation $y = A\cos\left(\frac{2t}{10}\right) + B\sin\left(\frac{2t}{10}\right)$.

a) Given that at time $t = 0$, $y(0) = 1.2$ and $\frac{\mathrm{d}y}{\mathrm{d}x}(0) = 0.5$ find the values of the constants A and B.

[3 marks]

b) Hence find the maximum displacement of the particle from the equilibrium point.

[4 marks]

13 Consider the parallelogram $OACB$ (labelled anticlockwise). Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Further, let M be a point that lies $\frac{1}{3}$ of the way along OA and N be a point that lies $\frac{2}{3}$ of the way along AC.

a) Find the vector $\overrightarrow{OC}$.

[1 mark]

b) Find the vector $\overrightarrow{MN}$.

[3 marks]

c) What can be deduced about the vectors $\overrightarrow{OC}$ and $\overrightarrow{MN}$ from parts (a) and (b)? And why?

[2 marks]

14 Consider the equation $e^x \sin(x - 3) + 1 = 0$.

a) Show that there is a root between 0 and 2 for the above equation.

[2 marks]

b) Use the Newton-Raphson process with $x_0 = 0.5$ to find the approximation x_1 to one of the roots of the equation above.

[3 marks]

c) Using your calculator to find a reference value for this root, calculate the percentage error made in using one step of the Newton-Raphson iteration.

[2 marks]

15 Find, in exact form $\tan(75°)$.

[6 marks]

16 Find the binomial expansion of $\dfrac{3x+5}{(1-x)(3x+1)}$ and state the range of values for which this expansion is valid.

[9 marks]

17 Find the area between the functions $f(x) = \cos(x)$ and $g(x) = \sin(x)$ in the interval $\left[\frac{-3\pi}{4}, \frac{\pi}{4}\right]$.

[4 marks]

18 It is given that $\theta = \arcsin(2x)$.

a) Writing $\sin(\theta) = 2x$, use implicit differentiation to show that

$\dfrac{d\theta}{dx} = \dfrac{A}{\sqrt{1 - Bx^2}}$ where A and B are integers to be found.

[3 marks]

b) Hence find the general solution of the differential equation

$$(1 - 4x^2)^{\frac{1}{2}} \frac{dy}{dx} = 2y$$

[5 marks]