AQA FM Paper 2 2019 “Predicted Content”

Name: ....

Class: ....

Mark: / 124

1 Calculate $\sum_{r=5}^{10} r(r+1)$

400 $\quad$ 420 $\quad$ 440 $\quad$ 532

[1 mark]

2 The vectors $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ a \\ 1 \end{pmatrix}$ are perpendicular. Find the value of a.

3 $\quad$ -3 $\quad$ 9 $\quad$ -9

[1 mark]

3 Let $z_1 = 2e^{i\frac{\pi}{3}}$ and $z_2 = z_2 = 3e^{i\frac{\pi}{12}}$. , if $-\pi < \arg(z) \leq \pi,\ \arg(z_1^3 z_2)$ is equal to

$-\dfrac{11\pi}{12}$ $\quad$ $\dfrac{13\pi}{12}$ $\quad$ $\dfrac{11\pi}{12}$ $\quad$ $\dfrac{7\pi}{36}$

[1 mark]

4 Show, with full reasoning, that $\int_{-\infty}^{\infty} x\mathrm{e}^{-x^2} \, \mathrm{d}x = 0$.

[5 marks]

5 **a)** Show that

$$\frac{1}{k+2} - \frac{1}{k+4} = \frac{2}{k^2 + 6k + 8}$$

[2 marks]

b) Hence, find an expression for

$$\sum_{k=1}^{n} \frac{2}{k^2 + 6k + 8}$$

[5 marks]

c) Using your result to part (b), or otherwise, evaluate

$$\sum_{k=1}^{\infty} \frac{2}{k^2 + 6k + 8}$$

[2 marks]

6 The circle $x^2 + y^2 = 1$ undergoes three transformations. It is translated by the vector $\begin{pmatrix} -\frac{2}{5} \\ \frac{3}{4} \end{pmatrix}$ before being stretched horizontally with a scale factor 5. Finally it is stretched vertically with a scale factor 4.

a) Find an equation for the resulting conic and determine its nature.

[5 marks]

b) Sketch the resulting conic.

[2 marks]

7 a) Prove that $\operatorname{arsinh}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$.

[4 marks]

b) Hence, find in logarithmic form the solutions to

$$\cosh^2(x) - 2\sinh(x) - 4 = 0$$

[4 marks]

8 **a)** Find a reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n(x)\ \mathrm{d}x$

[4 marks]

b) Hence, evaluate $\int_0^{\frac{\pi}{2}} \cos^2(x)(1+\cos^2(x))\ \mathrm{d}x$.

[4 marks]

9 Consider the function $y = \dfrac{x^2+2}{x}$.

a) Find the equations of the asymptotes to this curve.

[3 marks]

b) Find, using quadratic theory, the stationary points of the curve $y = \dfrac{x^2 + 2}{x}$. Solutions employing the calculus will not be awarded marks.

[5 marks]

c) Hence, sketch the curve $y = \dfrac{x^2 + 2}{x}$.

[3 marks]

10 **a)** Solve, analytically, the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x + y$$

where $y(1) = 4$.

[5 marks]

b) Use Euler's method with a step size of 0.1 to find an approximate value for $y(1.3)$.

[3 marks]

c) Consider the Improved Euler Method shown below

$$y_{r+1} = y_{r-1} + 2hf(x_r, y_r), \qquad x_{r+1} = x_r + h$$

Using the boundary value $y(1) = 4$ and the value calculated in part (b) for $y(1.1)$ find an approximate value for $y(1.3)$ using this Improved Euler Method.

[3 marks]

d) With reference to the exact solution found in part (a) comment on the accuracy of the Euler Method and the Improved Euler Method.

[2 marks]

11 Find

$$\int \frac{4x}{(x+1)(x^2+4)}\ \mathrm{d}x$$

[8 marks]

12 Evaluate $\lim\limits_{x \to 0} \dfrac{\arcsin(2x)}{\arctan(7x)}$.

[5 marks]

13 On an Argand diagram shade the region given by

$$\{z : |z - 1 + \mathrm{i}| < 3\} \cap \left\{ \frac{\pi}{6} \leq \arg\left(z - 1 + 2\mathrm{i}\right) \leq \frac{\pi}{3} \right\}$$

[5 marks]

14 Prove, by induction, that $3^{2n} + 7$ is divisible by 8.

[7 marks]

15 Sketch $f(x) = \left|(x+1)(x-2)(x+3)\right|$

[6 marks]

16 a) Find the invariant lines under the transformation

$$T = \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

[6 marks]

b) Describe the transformation T geometrically.

[2 marks]

17 **a)** Compute the arc length of $y = \ln\left(\sec(x)\right)$ between $x = 0$ and $x = \dfrac{\pi}{3}$.

[5 marks]

b) The curve described in part (a), when rotated around the $x-$axis forms the curved boundary of titanium paperweight that has been machined on a CNC lathe. Showing full working, approximately determine the volume of the paperweight by employing the mid-ordinate rule where the number of strips, n, is 3.

[4 marks]

18 Consider the matrix $A = \begin{pmatrix} 1 & 3 \\ -1 & 5 \end{pmatrix}$

a) Find the eigenvalues of A.

[3 marks]

b) Find the corresponding eigenvectors.

[4 marks]

c) Find the inverse matrix A^{-1} and verify that $\lambda_1 = \frac{1}{2}$ is an eigenvalue of A^{-1} with corresponding eigenvector $\mathbf{v_1} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

[3 marks]

d) Explain why 16 must be an eigenvector of A^4.

[2 marks]