# AQA A-Level Further Mathematics Warmup - Paper 1 2019

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| Define the exponential forms for the hyperbolic functions. | The quadratic equation $3x^2 + 10x + 5 = 0$ has roots $\alpha, \beta$ find the quadratic equation with roots $\frac{\alpha+1}{2}, \frac{\beta+1}{2}$. | How can you convert a polar coordinate $(r, \theta)$ into cartesian coordinates? | For the 2nd order ODE $a\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + b\frac{\mathrm{d}y}{\mathrm{d}x} + cy = 0$ describe how the discriminant of the auxiliary equation $am^2 + bm + c = 0$ determines the general solution. | State L'Hôpital's rule. |
| What are the $n$th roots of unity? | Define the vector product for two vectors $\mathbf{a}$ and $\mathbf{b}$. | What equation do the eigenvalues $\lambda$, and eigenvectors $\mathbf{v}$ of the matrix $M$ satisfy? How do you find eigenvalues? | State the modulus and argument form, and the exponential form of the complex number $z = a + \mathrm{i}b$. | What is the determinant and inverse of the $2 \times 2$ matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$? |
| Give the vector and cartesian equation of a line where $\mathbf{a}$ is a point on the line and $\mathbf{b}$ is the direction of the line. | What is $\sum_{i=1}^{n} r$ ? | Derive the logarithmic form of $\text{arcosh}(x)$ | State Viète's formulae for the cubic equation $ax^3 + bx^2 + cx + d = 0$ with roots $\alpha, \beta$ and $\gamma$. | Describe how to find the inverse of a $3 \times 3$ matrix $M = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$. |
| How could you use quadratic theory to find the turning points of $y = \frac{2x^2 + 2x - 3}{x - 3}$? | What are the key properties of Simple Harmonic Motion (SHM)? | What is the mean value of the function $f(x)$ over the interval $[a, b]$. | For the complex number $z = \cos(\theta) + \mathrm{i}\sin(\theta)$ state the relations relating powers of $z$ and $\theta$. | In the context of hyperbolic functions describe Osborn's rule and the affect this has on the identity $\cos^2(x) + \sin^2(x) = 1$ |
| How do you diagonalise the matrix $M$? | Give a suitable concluding statement for a proof by induction. | What is the difference between a line of invariant points and an invariant line for the matrix $M$? | Define the scalar product for two vectors $\mathbf{a}$ and $\mathbf{b}$. | How does the discriminant of the auxiliary equation for damped harmonic motion determine the type of damping? |

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| $\cosh(x) = \frac{e^x + e^{-x}}{2}$<br>$\sinh(x) = \frac{e^x - e^{-x}}{2}$<br>$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$ | Use the substitution $x = 2w - 1$ to obtain $12w^2 + 8w - 2$. | Use $x^2 + y^2 = r^2$, $x = r\cos(\theta)$, $y = r\sin(\theta)$ and $\tan(\theta) = \frac{y}{x}$ | $b^2 - 4ac > 0$, distinct real roots $\alpha, \beta$ so $y = Ae^{\alpha x} + Be^{\beta x}$.<br>$b^2 - 4ac = 0$, repeated real root $\alpha$ so $y = (A + Bx)e^{\alpha x}$.<br>$b^2 - 4ac < 0$, complex roots $p \pm q\mathrm{i}$ so $y = e^{px}(A\cos(qx) + B\sin(qx))$ | If $\lim_{x\to c} f(x)$ ad $\lim_{x\to c} g(x)$ are both zero or $\pm\infty$ then $\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$ |
| $\omega = e^{\mathrm{i}\frac{2k\pi}{n}}$ | $\mathbf{a}\times\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin(\theta)\hat{\mathbf{n}}$<br>$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ | $M\mathbf{v} = \lambda\mathbf{v}$.<br>Solve $\det(M - \lambda I) = 0$. | $z = re^{\mathrm{i}\theta}$ and $z = r(\cos(\theta) + \mathrm{i}\sin(\theta))$ where $r$ is the modulus of $z$ and $\theta$ is its argument. | $\det(A) = \lvert A\rvert = ad - bc$<br>$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ |
| $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$<br>$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} = c$<br>where $c$ is a constant. | $\sum_{i=1}^{n} r = \frac{1}{2}n(n+1)$ | $\text{arcosh}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$ | $\alpha + \beta + \gamma = \frac{-b}{a}$<br>$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$<br>$\alpha\beta\gamma = -\frac{d}{a}$ | Find the determinant of the matrix. Find the matrix of minors $\begin{pmatrix} A & B & C \\ D & E & F \\ G & H & I \end{pmatrix}$, cofactors $\begin{pmatrix} A & -B & C \\ -D & E & -F \\ G & -H & I \end{pmatrix}$. Transpose this and multiply by the reciprocal of the determinant to get the inverse. |
| Set $k = \frac{2x^2 + 2x - 3}{x - 3}$, rearrange to form a quadratic and find the discriminant. Set equal to zero to find the y co-ordinates of the turning points. | $\ddot{x} = -\omega^2 x$. Period $\frac{2\pi}{\omega}$.<br>The force acting on the object undergoing SHM is proportional to its displacement but in the opposite direction. The general solution $x = A\cos(\omega t) + B\sin(\omega t)$ can be rewritten in the form $x = R\cos(\omega t - \phi)$. | $M = \frac{1}{b - a}\int_a^b f(x)\,\mathrm{d}x$ | $\frac{z^n + z^{-n}}{2} = \cos(n\theta)$<br>$2\cos(n\theta) = z^n + \frac{1}{z^n}$<br>$\frac{z^n - z^{-n}}{2\mathrm{i}} = \sin(n\theta)$<br>$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin(n\theta)$ | For every product or implied product of sines the sign is changed.<br>$\cosh^2(x) - \sinh^2(x) = 1$ |
| Find the eigenvalues and eigenvectors of the matrix. Then $M = UDU^{-1}$ where $D$ is a diagonal matrix containing the eigenvalues and $U$ is a matrix whose columns as the (distinct) eigenvectors of $M$. | As the statement is true for $n = 1$ and we have shown that if it holds for $n = k$ then it also holds for $n = k + 1$ we conclude that the statement must be true for all $n \geq 1$ by the principle of mathematical induction. | Invariant points map to themselves under the action of $M$, so a line of invariant points is a line of such points.<br>The image of any point on an invariant line remains on the line but is not necessarily the original point. | $\mathbf{a}\cdot\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos(\theta)$<br>$\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ | Discriminant less than zero means the system is lightly damped.<br>Discriminant equal to zero means the system is critically damped.<br>Discriminant greater than zero is heavily damped. |