Year 13 Maths - Exam Question-a-Day Revision Plan

Aims:

The idea of this plan is that it provides an exam question for you to do every day in the run up to your exams.

These questions could be used as a starter for some more general revision that day.

The topics for the questions each day will cycle allowing revision of every topic to take place. The first couple of weeks will contain questions taken from AS papers and after that questions from both AS and full A-Level papers will be used to ensure broad coverage of the syllabus.

February 2019

Day	Date	Topic
1	1st February	Integration of Polynomials
2	2nd February	Decreasing Functions
3	3rd February	Straight Lines
4	4th February	Solving Trigonometric Equations
5	5th February	Simultaneous Equations
6	6th February	Factor Theorem
7	7th February	Algebraic Proof
8	8th February	Vector Geometry
9	9th February	Non Right Angle Trigonometry
10	10th February	Velocity-Time Graphs
11	11th February	Using the Discriminant
12	12th February	Discrete Probability
13	13th February	Completing the Square
14	14th February	Kinematics
15	15th February	Graph Transformations
16	16th February	Surds
17	17th February	Perpendicular Lines
18	18th February	Exponential Models
19	19th February	Inverse Functions
20	20th February	Equations of Tangents
21	21st February	Venn Diagrams
22	22nd February	Factor Theorem
23	23rd February	Binomial Expansion
24	24th February	Quadratics and Graph Transformations
25	25th February	Differentiation and Tangents
26	26th February	Iterative Methods
27	27th February	Counter Examples & Direct Proof
28	28th February	Connected Rates of Change

March 2019

Day	Date	Topic
1	1st March	Differentiation of Simple Functions
2	2nd March	Turning Points and Transformations
3	3rd March	Vectors (with Modelling)
4	4th March	Simultaneous Equations
5	5th March	Modelling with Differential Equations
6	6th March	Projectiles
7	7th March	Logarithms
8	8th March	Small Angle Approximations
9	9th March	Trapezium Rule and Integration
10	10th March	Integration by Parts
11	11th March	Parametric Equations
12	12th March	Newton-Raphson Method
13	13th March	Factor Theorem
14	14th March	Inequality
15	15th March	Integration
16	16th March	Resolving Forces
17	17th March	Modelling and Maximisation
18	18th March	Arc Length and Area of Sectors
19	19th March	Function Notation
20	20th March	Trigonometric Identities
21	21st March	Calculus and Maximisation
22	22nd March	Stationary Points
23	23rd March	Geometric Series
24	24th March	Graph Transformations
25	25th March	Differentiation from First Principles
26	26th March	Modelling and Exponential Functions
27	27th March	Proof by Contradiction
28	28th March	Moments
29	29th March	Differentiation
30	30th March	Logarithmic Models
31	31st March	Differential Equations

April 2019

Day	Date	Topic
1	1st April	Validity of binomial expansions.
2	2nd April	Use of factor theorem.
3	3rd April	Stationary points.
4	4th April	Parametric equations.
5	5th April	Solving trigonometric equations.
6	6th April	Graph transformations and Rcos(alpha-a) form.
7	7th April	Differential equations.
8	8th April	Function notation.
9	9th April	Integration by substitution.
10	10th April	Proof.
11	11th April	Hypothesis testing.
12	12th April	Straight lines.
13	13th April	Inflection points.
14	14th April	Differential equations.
15	15th April	Moments.
16	16th April	Normal distribution.
17	17th April	Geometric series.
18	18th April	Interpreting graphical displays of data.
19	19th April	Deriving SUVAT equations.
20	20th April	Venn diagrams and probability.
21	21st April	Minimisation by calculus.
22	22nd April	Implicit differentiation and stationary points.
23	23rd April	Velocity-time graphs.
24	24th April	Proof by contradiction.
25	25th April	Always, sometimes, never and reasoning.
26	26th April	Motion with respect to time.
27	27th April	Mathematical Modelling.
28	28th April	Scatter plots.
29	29th April	Use of normal distribution.
30	30th April	Arithmetic sequences.

May 2019

Day	Date	Topic
1	1st May	Solving Inequalities.
2	2nd May	Arc length and area of sectors.
3	3rd May	Algebraic fractions.
4	4th May	Histograms and Normal models.
5	5th May	Hypothesis testing.
6	6th May	Trapezium rule.
7	7th May	Integration under a curve.
8	8th May	Linear regression.
9	9th May	Rcos(x-alpha) form.
10	10th May	Stationary points.
11	11th May	Integration.
12	12th May	Variable acceleration.
13	13th May	Binomial expansion.
14	14th May	Points of inflection.
15	15th May	Methods of integration.
16	16th May	Concave functions and trapezium rule.
17	17th May	Factor theorem and polynomials.
18	18th May	Binomial expansion.
19	19th May	Small angle approximations.
20	20th May	Rough planes.
21	21st May	Integration of rational functions.
22	22nd May	Collinearity,
23	23rd May	Curves and equations of normals.
24	24th May	Stationary points, sequences, domain.
25	25th May	Solving trigonometric equations.
26	26th May	Sampling.
27	27th May	Projectiles.
28	28th May	Forces on rough planes.
29	29th May	Modelling.
30	30th May	Non-constant acceleration (vector form).
31	31st May	Binomial distribution.

June 2019

Day	Date	Topic
1	1st June	Trigonometry.
2	2nd June	Exponential models.
3	3rd June	Projectiles.
4	4th June	Conditional probability.
5	5th June	Disproof by counter-example.
6	6th June	Interpreting scatter diagrams.
7	7th June	Connected particles.
8	8th June	Discrete distributions.
9	9th June	Use of summation notation and logarithm laws.
10	10th June	Resultant forces
11	11th June	Hypothesis testing.
12	12th June	Hypothesis testing.
13	13th June	Summary statistics.

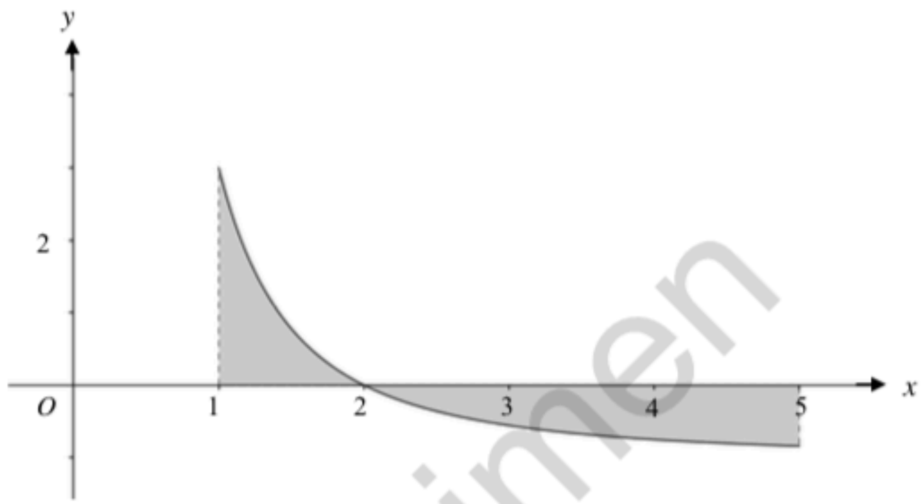
February Questions

1st February

5 **(a)** Find $\int (x^3 - 6x)\,dx$. **[3]**

(b) **(i)** Find $\int \left(\frac{4}{x^2} - 1\right)dx$. **[3]**

(ii) The diagram shows part of the curve $y = \frac{4}{x^2} - 1$.



The curve crosses the x-axis at (2, 0).
The shaded region is bounded by the curve, the x-axis, and the lines $x = 1$ and $x = 5$.

Calculate the area of the shaded region. **[3]**

OCR AS Level Mathematics (H230) SAMS, Paper 1

2nd February

8 Prove that the function $f(x) = x^3 - 3x^2 + 15x - 1$ is an increasing function.

[6 marks]

AQA AS SAMS Paper 2

3rd February

1. The line l passes through the points A (3, 1) and B (4, −2).

Find an equation for l.

(3)

Pearson Edexcel SAMS AS Paper 1

4th February

5 Jessica, a maths student, is asked by her teacher to solve the equation $\tan x = \sin x$, giving all solutions in the range $0° \le x \le 360°$

The steps of Jessica's working are shown below.

		$\tan x = \sin x$	
Step 1	$\Rightarrow$	$\dfrac{\sin x}{\cos x} = \sin x$	Write $\tan x$ as $\dfrac{\sin x}{\cos x}$
Step 2	$\Rightarrow$	$\sin x = \sin x \cos x$	Multiply by $\cos x$
Step 3	$\Rightarrow$	$1 = \cos x$	Cancel $\sin x$
	$\Rightarrow$	$x = 0°$ or $360°$	

The teacher tells Jessica that she has not found all the solutions because of a mistake.

Explain why Jessica's method is not correct.

[2 marks]

AQA AS SAMS Paper 1

5th February

11 **In this question you must show detailed reasoning.**

Determine for what values of k the graphs $y = 2x^2 - kx$ and $y = x^2 - k$ intersect. **[6]**

OCR AS Level Mathematics (H630) SAMS Paper 1

6th February

4.

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

Pearson Edexcel SAMS AS Paper 1

7th February

12 (a) Given that n is an even number, prove that $9n^2 + 6n$ has a factor of 12

[3 marks]

12 (b) Determine if $9n^2 + 6n$ has a factor of 12 for any integer n .

[1 mark]

AQA AS SAMS Paper 2

8th February

4 The points A, B and C have position vectors $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ respectively.

M is the midpoint of BC.

(a) Find the position vector of the point D such that $\overrightarrow{BC} = \overrightarrow{AD}$. **[3]**

(b) Find the magnitude of $\overrightarrow{AM}$. **[3]**

9th February

8.

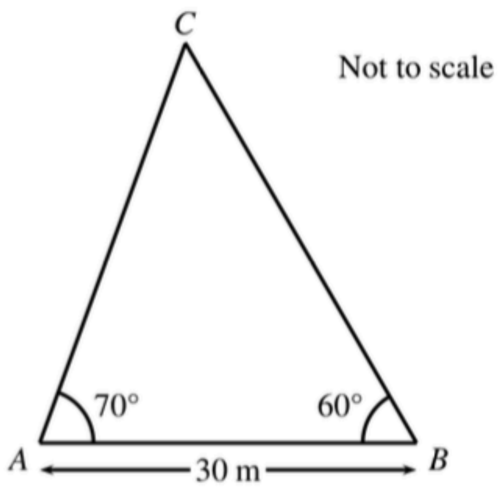


Figure 1

A triangular lawn is modelled by the triangle ABC, shown in Figure 1. The length AB is to be 30 m long.

Given that angle $BAC = 70°$ and angle $ABC = 60°$,

(a) calculate the area of the lawn to 3 significant figures.

(4)

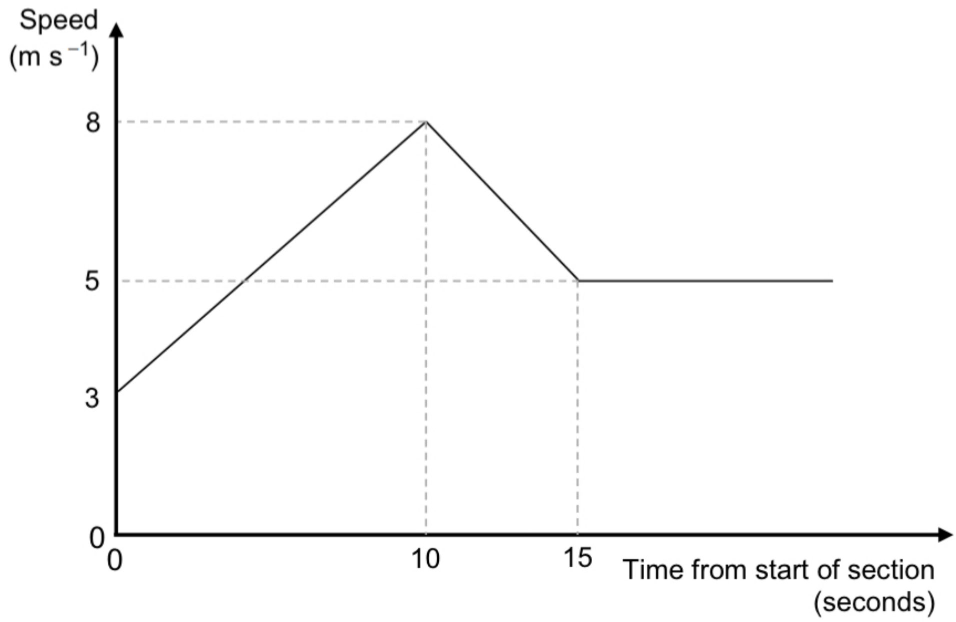
(b) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

Pearson Edexcel SAMS AS Paper 1

10th February

15 The graph shows how the speed of a cyclist varies during a timed section of length 120 metres along a straight track.



15 (a) Find the acceleration of the cyclist during the first 10 seconds.

[1 mark]

15 (b) After the first 15 seconds, the cyclist travels at a constant speed of 5 m s^{-1} for a further T seconds to complete the 120-metre section.

Calculate the value of T.

[4 marks]

AQA AS SAMS Paper 1

11th February

10. The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

(4)

Pearson Edexcel SAMS AS Paper 1

12th February

9 The probability distribution of a random variable X is given in the table.

x	1	2	3
$\mathrm{P}(X = x)$	0.6	0.3	0.1

Two values of X are chosen at random.

Find the probability that the second value is greater than the first. **[3]**

OCR AS Level Mathematics (H230) SAMS Paper 1

13th February

4 **(a)** Express $x^2 + 4x + 7$ in the form $(x+b)^2 + c$. **[2]**

(b) Explain why the minimum point on the curve $y = (x+b)^2 + c$ occurs when $x = -b$. **[1]**

OCR AS Level Mathematics (H630) SAMS Paper 1

14th February

16 A particle, of mass 400 grams, is initially at rest at the point O.

The particle starts to move in a straight line so that its velocity, v m s^{-1}, at time t seconds is given by

$$v = 6t^2 - 12t^3 \text{ for } t > 0$$

16 (a) Find an expression, in terms of t, for the force acting on the particle.

[3 marks]

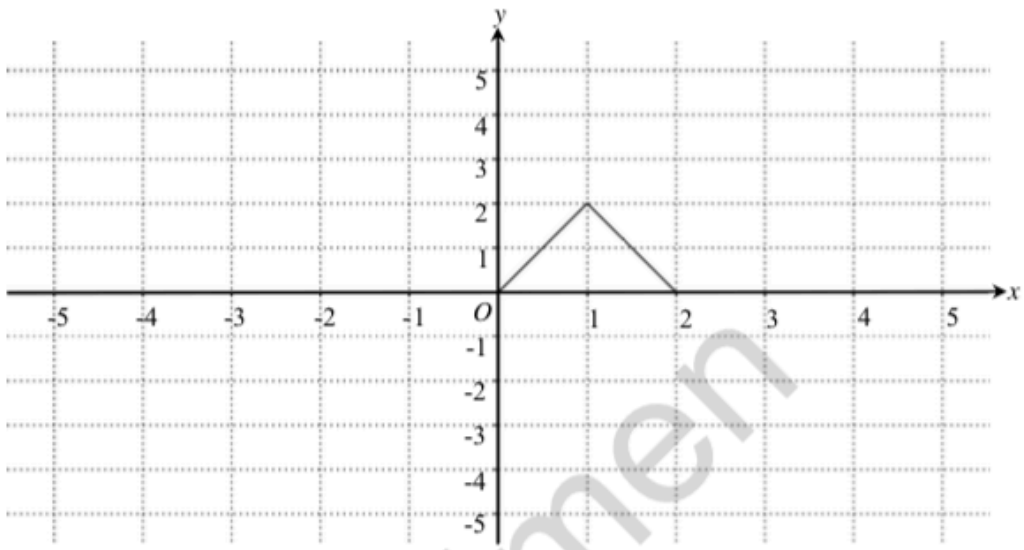
16 (b) Find the time when the particle next passes through O.

[5 marks]

AQA AS Mathematics SAMS Paper 1

15th February

1 **(a)** The diagram below shows the graph of $y = \mathrm{f}(x)$.



(i) On the diagram in the Printed Answer Booklet draw the graph of $y = \mathrm{f}(x+3)$. **[2]**

(ii) Describe fully the transformation which transforms the graph of $y = \mathrm{f}(x)$ to the graph of $y = -\mathrm{f}(x)$. **[1]**

(b) The point (2, 3) lies on the graph of $y = \mathrm{g}(x)$.

State the coordinates of its image when $y = \mathrm{g}(x)$ is transformed to

(i) $y = 4\mathrm{g}(x)$ **[1]**

(ii) $y = \mathrm{g}(4x)$. **[1]**

OCR AS Level Mathematics (H230) SAMS Paper 2

16th February

1 Simplify fully.

(a) $\sqrt{a^3} \times \sqrt{16a}$ **[2]**

(b) $(4b^6)^{\frac{5}{2}}$ **[2]**

OCR A-Level Mathematics (H240) SAMS Paper 1

17th February

7 Determine whether the line with equation $2x + 3y + 4 = 0$ is parallel to the line through the points with coordinates (9, 4) and (3, 8).

[4 marks]

AQA A-Level Mathematics SAMS Paper 1

18th February

14.

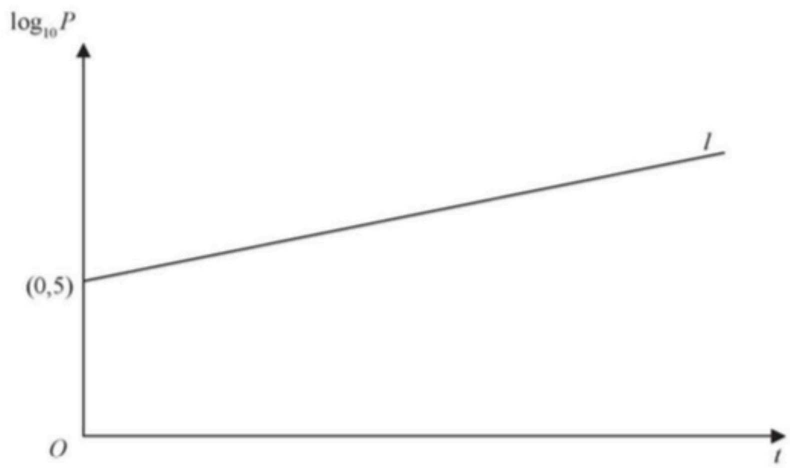


Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10} P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l.

(2)

(b) Find the value of a and the value of b.

(4)

(c) With reference to the model interpret

(i) the value of the constant a,

(ii) the value of the constant b

(2)

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(e) State two reasons why this may not be a realistic population model.

(2)

Pearson Edexcel SAMS AS Paper 1

19th February

11 For all real values of x, the functions f and g are defined by $\mathrm{f}(x) = x^2 + 8ax + 4a^2$ and $\mathrm{g}(x) = 6x - 2a$, where a is a positive constant.

(a) Find $\mathrm{fg}(x)$.
Determine the range of $\mathrm{fg}(x)$ in terms of a. **[4]**

(b) If $\mathrm{fg}(2) = 144$, find the value of a. **[3]**

(c) Determine whether the function fg has an inverse. **[2]**

OCR A-Level Mathematics (H240) SAMS Paper 1

20th February

3. A curve has the equation $y = \ln 3x - \mathrm{e}^{-2x}$.

Show that the equation of the tangent at the point with an x-coordinate of 1 is

$$y = \left(\frac{\mathrm{e}^2 + 2}{\mathrm{e}^2}\right)x - \left(\frac{\mathrm{e}^2 + 3}{\mathrm{e}^2}\right) + \ln 3.$$

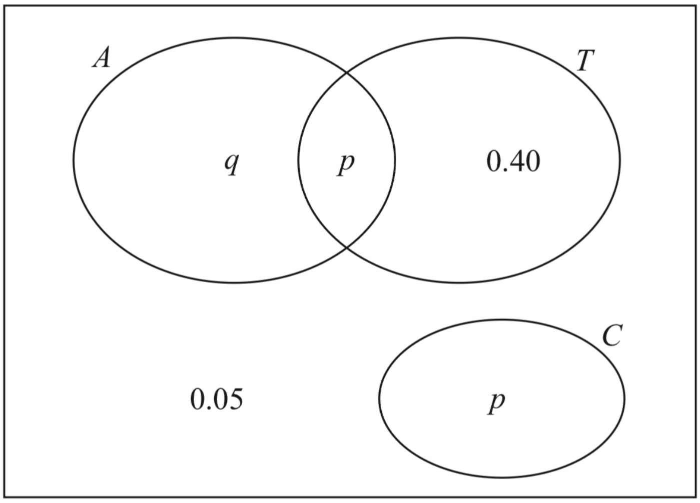
(6 marks)

Pearson Edexcel A-Level Mathematics Practice Paper B

21st February

3. The Venn diagram shows the probabilities for students at a college taking part in various sports.

A represents the event that a student takes part in Athletics.
T represents the event that a student takes part in Tennis.
C represents the event that a student takes part in Cricket.
p and q are probabilities.



The probability that a student selected at random takes part in Athletics or Tennis is 0.75

(a) Find the value of p.

(1)

(b) State, giving a reason, whether or not the events A and T are statistically independent. Show your working clearly.

(3)

(c) Find the probability that a student selected at random does not take part in Athletics or Cricket.

(1)

22nd February

1 $\mathrm{p}(x) = x^3 - 5x^2 + 3x + a$, where a is a constant.

Given that $x - 3$ is a factor of $\mathrm{p}(x)$, find the value of a

Circle your answer.

[1 mark]

-9 $\quad$ -3 $\quad$ 3 $\quad$ 9

AQA AS-Level Mathematics SAMS Paper 2

23rd February

5 **(a)** Find the first three terms in the expansion of $(1+px)^{\frac{1}{3}}$ in ascending powers of x. **[3]**

(b) The expansion of $(1+qx)(1+px)^{\frac{1}{3}}$ is $1 + x - \frac{2}{9}x^2 + \dots$.

Find the possible values of the constants p and q. **[5]**

OCR A-Level Mathematics (H240) SAMS Paper 3

24th February

4.

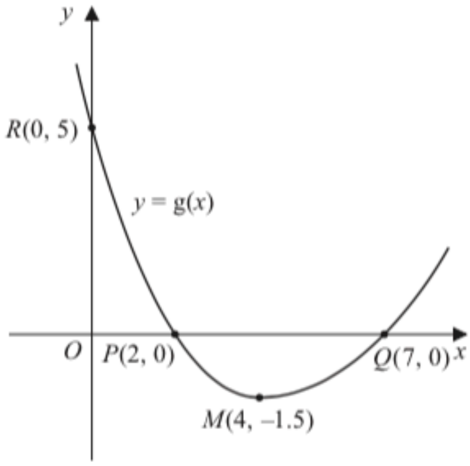


Figure 1

Figure 1 shows a sketch of the curve with equation $y = g(x)$.

The curve has a single turning point, a minimum, at the point $M(4, -1.5)$.

The curve crosses the x-axis at two points, $P(2, 0)$ and $Q(7, 0)$.

The curve crosses the y-axis at a single point $R(0, 5)$.

(a) State the coordinates of the turning point on the curve with equation $y = 2g(x)$.

(1)

(b) State the largest root of the equation

$$g(x + 1) = 0$$

(1)

(c) State the range of values of x for which $g'(x) \leqslant 0$

(1)

Given that the equation $g(x) + k = 0$, where k is a constant, has no real roots,

(d) state the range of possible values for k.

(1)

Pearson AS Mathematics Specimen Papers Paper 1

25th February

12 A curve has equation $y = 6x\sqrt{x} + \dfrac{32}{x}$ for $x > 0$

12 (a) Find $\dfrac{dy}{dx}$

[4 marks]

12 (b) The point *A* lies on the curve and has x-coordinate 4

Find the coordinates of the point where the tangent to the curve at *A* crosses the x-axis.

[5 marks]

AQA AS-Level SAMS Paper 1

26th February

9 The equation $x^3 - x^2 - 5x + 10 = 0$ has exactly one real root α.

(a) Show that the Newton-Raphson iterative formula for finding this root can be written as

$$x_{n+1} = \frac{2x_n^{\ 3} - x_n^{\ 2} - 10}{3x_n^{\ 2} - 2x_n - 5}.$$ **[3]**

(b) Apply the iterative formula in part (a) with initial value $x_1 = -3$ to find x_2, x_3, x_4 correct to 4 significant figures. **[1]**

(c) Use a change of sign method to show that $\alpha = -2.533$ is correct to 4 significant figures. **[3]**

(d) Explain why the Newton-Raphson method with initial value $x_1 = -1$ would not converge to α. **[2]**

OCR A-Level Mathematics (H240) SAMS Paper 3

27th February

6. (i) Use a counter example to show that the following statement is false.

"$n^2 - n - 1$ is a prime number, for $3 \leqslant n \leqslant 10$."

(2)

(ii) Prove that the following statement is always true.

"The difference between the cube and the square of an odd number is even."

For example $5^3 - 5^2 = 100$ is even.

(4)

Pearson AS Mathematics Specimen Papers Paper 1

28th February

11 The circle with equation $(x-7)^2+(y+2)^2=5$ has centre C.

11 (a) (i) Write down the radius of the circle.

[1 mark]

11 (a) (ii) Write down the coordinates of C.

[1 mark]

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11 (b) The point P (5, −1) lies on the circle.

Find the equation of the tangent to the circle at P, giving your answer in the form $y=mx+c$

[4 marks]

11 (c) The point Q (3, 3) lies outside the circle and the point T lies on the circle such that QT is a tangent to the circle. Find the length of QT.

[4 marks]

AQA AS-Level SAMS Paper 2

March Questions

1st March

2 A curve has equation $y = \dfrac{2}{\sqrt{x}}$

Find $\dfrac{dy}{dx}$

Circle your answer.

[1 mark]

$\dfrac{\sqrt{x}}{3}$ $\qquad$ $\dfrac{1}{x\sqrt{x}}$ $\qquad$ $-\dfrac{1}{x\sqrt{x}}$ $\qquad$ $-\dfrac{1}{2x\sqrt{x}}$

AQA A-Level Mathematics SAMS Paper 1

2nd March

15.

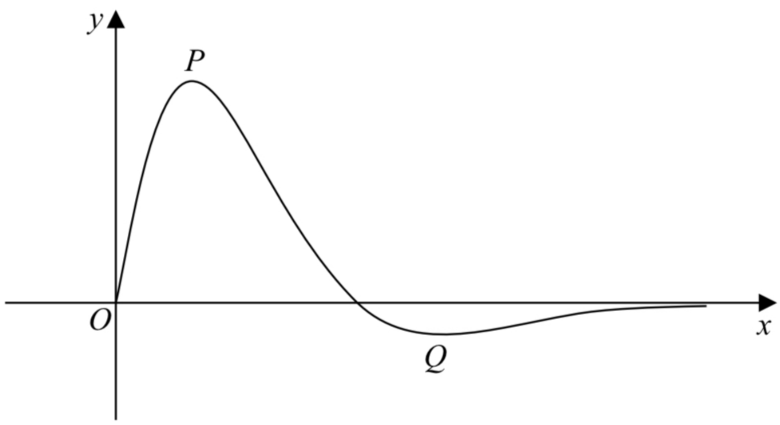


Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{4\sin 2x}{e^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{f}(2x)$.

(ii) $y = 3 - 2\mathrm{f}(x)$.

(4)

Pearson Edexcel A-Level Mathematics SAMS Paper 1

3rd March

12 A model boat has velocity $\mathbf{v} = \left((2t-2)\mathbf{i} + (2t+2)\mathbf{j}\right)$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north respectively and t is the time in seconds, where $t \geq 0$. The position vector of the boat is $(3\mathbf{i} + 14\mathbf{j})$ m when $t = 3$.

(i) Show that the boat is never instantaneously at rest. **[2]**

(ii) Determine any times at which the boat is moving directly northwards. **[2]**

(iii) Determine any times at which the boat is north-east of the origin. **[5]**

OCR B (H640) A-Level Mathematics SAMS Paper 1

4th March

1 Solve the simultaneous equations.

$$\begin{aligned} x^2 + 8x + y^2 &= 84 \\ x - y &= 10 \end{aligned}$$

[4]

OCR A (H240) A-Level Mathematics SAMS Paper 1

5th March

6 Sam goes on a diet. He assumes that his mass, m kg after t days, decreases at a rate that is inversely proportional to the cube root of his mass.

6 (a) Construct a differential equation involving m, t and a positive constant k to model this situation.

[3 marks]

6 (b) Explain why Sam's assumption may not be appropriate.

[1 mark]

AQA A-Level Mathematics SAMS Paper 1

6th March

7 **In this question take $g = 10$.**

A small stone is projected from a point O with a speed of $26\ \mathrm{m\,s^{-1}}$ at an angle θ above the horizontal. The initial velocity and part of the path of the stone are shown in Fig. 7. You are given that $\sin\theta = \frac{12}{13}$. After t seconds the horizontal and vertical displacements of the stone from O are x metres and y metres.

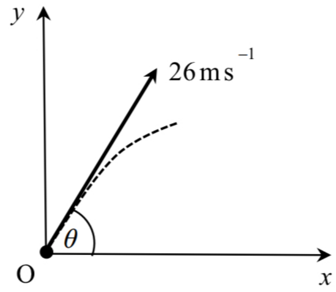


Fig. 7

(i) Using the standard model for projectile motion,

- show that $y = 24t - 5t^2$,
- find an expression for x in terms of t. **[4]**

The stone passes through a point A which is $16\ \mathrm{m}$ above the level of O.

(ii) Find the two possible horizontal distances of A from O. **[4]**

Suppose that a toy balloon is projected from O with the same initial velocity as the small stone.

(iii) Suggest two ways in which the standard model could be adapted. **[2]**

OCR B (H640) A-Level Mathematics SAMS Paper 1

7th March

5 **In this question you must show detailed reasoning.**

Use logarithms to solve the equation

$$3^{2x+1} = 4^{100},$$

giving your answer correct to 3 significant figures. **[4]**

OCR A (H240) A-Level Mathematics SAMS Paper 1

8th March

3 When θ is small, find an approximation for $\cos 3\theta + \theta \sin 2\theta$, giving your answer in the form $a + b\theta^2$

[3 marks]

AQA A-Level Mathematics SAMS Paper 1

9th March

14.

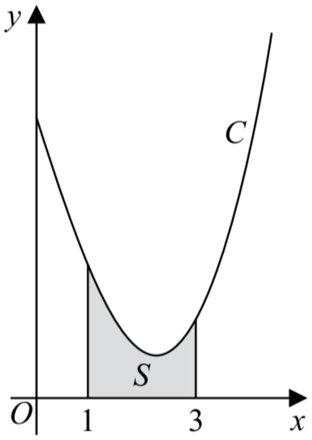


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(6)

(In part c, solutions based entirely on graphical or numerical methods are not acceptable.)

10th March

8 Find $\int x^2 e^{2x}\, dx$. **[7]**

OCR B (H640) A-Level Mathematics SAMS Paper 1

11th March

13. The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(2)

The point P lies on C where $t = \frac{2\pi}{3}$

The line l is the normal to C at P.

(b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.

You must show clearly how you obtained your answers.

(6)

Pearson Edexcel A-Level Mathematics SAMS Paper 1

12th March

9 The equation $x^3 - x^2 - 5x + 10 = 0$ has exactly one real root α.

(i) Show that the Newton-Raphson iterative formula for finding this root can be written as

$$x_{n+1} = \frac{2x_n^{\ 3} - x_n^{\ 2} - 10}{3x_n^{\ 2} - 2x_n - 5}.$$

[3]

(ii) Apply the iterative formula in part **(i)** with initial value $x_1 = -3$ to find x_2, x_3, x_4 correct to 4 significant figures. **[1]**

(iii) Use a change of sign method to show that $\alpha = -2.533$ is correct to 4 significant figures. **[3]**

(iv) Explain why the Newton-Raphson method with initial value $x_1 = -1$ would not converge to α. **[2]**

OCR A (H240) A-Level Mathematics SAMS Paper 1

13th March

4 $\mathrm{p}(x) = 2x^3 + 7x^2 + 2x - 3$

4 (a) Use the factor theorem to prove that $x+3$ is a factor of $\mathrm{p}(x)$

[2 marks]

4 (b) Simplify the expression $\dfrac{2x^3 + 7x^2 + 2x - 3}{4x^2 - 1}$, $x \neq \pm\dfrac{1}{2}$

[4 marks]

AQA A-Level Mathematics SAMS Paper 1

14th March

3 Solve the inequality $|2x-1| \geq 4$. **[4]**

OCR A (H260) A-Level Mathematics SAMS Paper 1

15th March

Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,\mathrm{d}t = \ln 7$$

show that $a = \ln k$, where k is a constant to be found.

(4)

Pearson Edexcel A-Level Mathematics SAMS Paper 1

16th March

5 Dora is trying to pull a loaded sledge along horizontal ground. The only resistance to motion of the sledge is due to friction between it and the ground.

Fig. 5

Initially she pulls with a force of 100 N inclined at 32° to the horizontal, as shown in Fig.5, but the sledge does not move.

(i) Determine the frictional force between the ground and the sledge. Give your answer correct to 3 significant figures. **[2]**

(ii) Next she pulls with a force of 100 N inclined at a smaller angle to the horizontal. The sledge still does not move. Compare the frictional force in this new situation with that in part **(i)**, justifying your answer. **[2]**

OCR B(H640) A-Level Mathematics SAMS Paper 1

17th March

14 An open-topped fish tank is to be made for an aquarium.

It will have a square horizontal base, rectangular vertical sides and a volume of 60 m^3

The materials cost:

- £15 per m^2 for the base
- £8 per m^2 for the sides.

14 (a) Modelling the sides and base of the fish tank as laminae, use calculus to find the height of the tank for which the overall cost of the materials has its minimum value.

Fully justify your answer.

[8 marks]

14 (b) (i) In reality, the thickness of the base and sides of the tank is 2.5 cm

Briefly explain how you would refine your modelling to take account of the thickness of the sides and base of the tank of the tank.

[1 mark]

14 (b) (ii) How would your refinement affect your answer to part **(a)**?

[1 mark]

AQA A-Level Mathematics SAMS Paper 1

18th March

2.

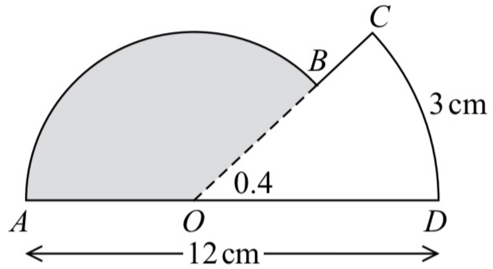


Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, $\angle COD$ = 0.4 radians and *AOD* is a straight line of length 12 cm,

(a) find the length of *OD*,

(2)

(b) find the area of the shaded sector *AOB*.

(3)

Pearson Edexcel A-Level Mathematics SAMS Paper 1

19th March

11 For all real values of x, the functions f and g are defined by $\mathrm{f}(x) = x^2 + 8ax + 4a^2$ and $\mathrm{g}(x) = 6x - 2a$, where a is a positive constant.

(i) Find $\mathrm{fg}(x)$. Determine the range of $\mathrm{fg}(x)$ in terms of a. **[4]**

(ii) If $\mathrm{fg}(2) = 144$, find the value of a. **[3]**

(iii) Determine whether the function fg has an inverse. **[2]**

OCR A (H240) A-Level Mathematics SAMS Paper 1

20th March

13 Prove the identity $\cot^2\theta - \cos^2\theta \equiv \cot^2\theta\cos^2\theta$

[3 marks]

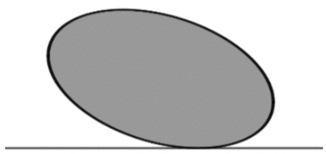
AQA A-Level Mathematics SAMS Paper 1

21st March

12 A sculpture formed from a prism is fixed on a horizontal platform, as shown in the diagram.

The shape of the cross-section of the sculpture can be modelled by the equation $x^2 + 2xy + 2y^2 = 10$, where x and y are measured in metres.

The x and y axes are horizontal and vertical respectively.



Find the maximum vertical height above the platform of the sculpture.

[8 marks]

AQA A-Level Mathematics SAMS Paper 1

22nd March

1. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

Pearson Edexcel A-Level Mathematics SAMS Paper 1

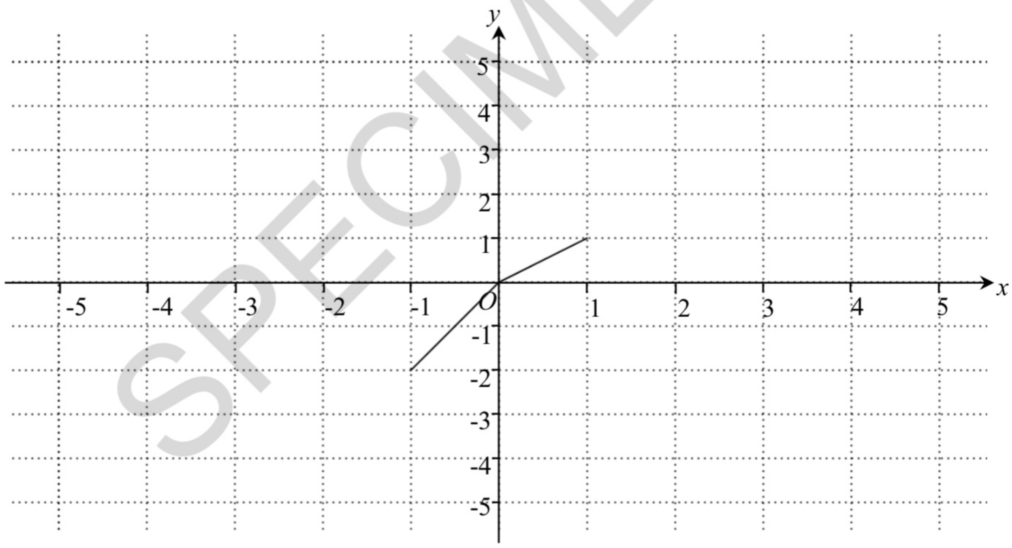
23rd March

2 A geometric series has first term 3. The sum to infinity of the series is 8. Find the common ratio. **[3]**

OCR B(H640) A-Level Mathematics SAMS Paper 1

24th March

3 The diagram below shows the graph of $y = \mathrm{f}(x)$.



(i) On the diagram in your Printed Answer Booklet, draw the graph of $y = \mathrm{f}(\frac{1}{2}x)$. **[1]**

(ii) On the diagram in your Printed Answer Booklet, draw the graph of $y = \mathrm{f}(x-2)+1$. **[2]**

OCR A (H240) A-Level Mathematics SAMS Paper 1

25th March

17 $\mathrm{f}(x) = \sin x$

Using differentiation from first principles find the exact value of $\mathrm{f}'\left(\frac{\pi}{6}\right)$

Fully justify your answer.

[6 marks]

AQA A-Level Mathematics SAMS Paper 1

26th March

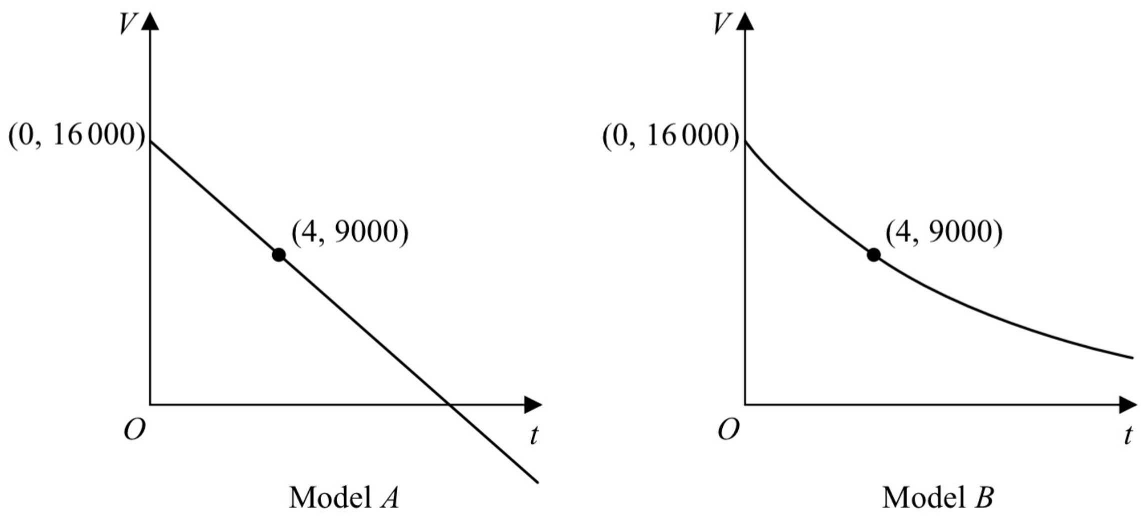
6. A company plans to extract oil from an oil field.

The daily volume of oil V, measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model A.

(2)

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model B.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(5)

27th March

6 Prove by contradiction that there is no greatest even positive integer. **[3]**

OCR A (H240) A-Level Mathematics SAMS Paper 1

28th March

15 Fig. 15 shows a uniform shelf AB of weight W N which is 180 cm long and rests on supports at points C and D. C is 30 cm from A and D is 60 cm from B.

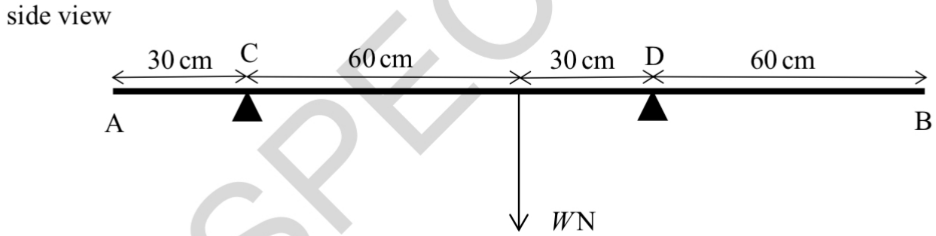


Fig. 15

Determine the range of positions a point load of 3 W could be placed on the shelf without it tipping. **[6]**

OCR A (H260) A-Level Mathematics SAMS Paper 1

29th March

10 A curve has equation $x = (y+5)\ln(2y-7)$.

(a) Find $\frac{dx}{dy}$ in terms of y. **[3]**

(b) Find the gradient of the curve where it crosses the y-axis. **[5]**

OCR A (H240) A-Level Mathematics SAMS Paper 1

30th March

12. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

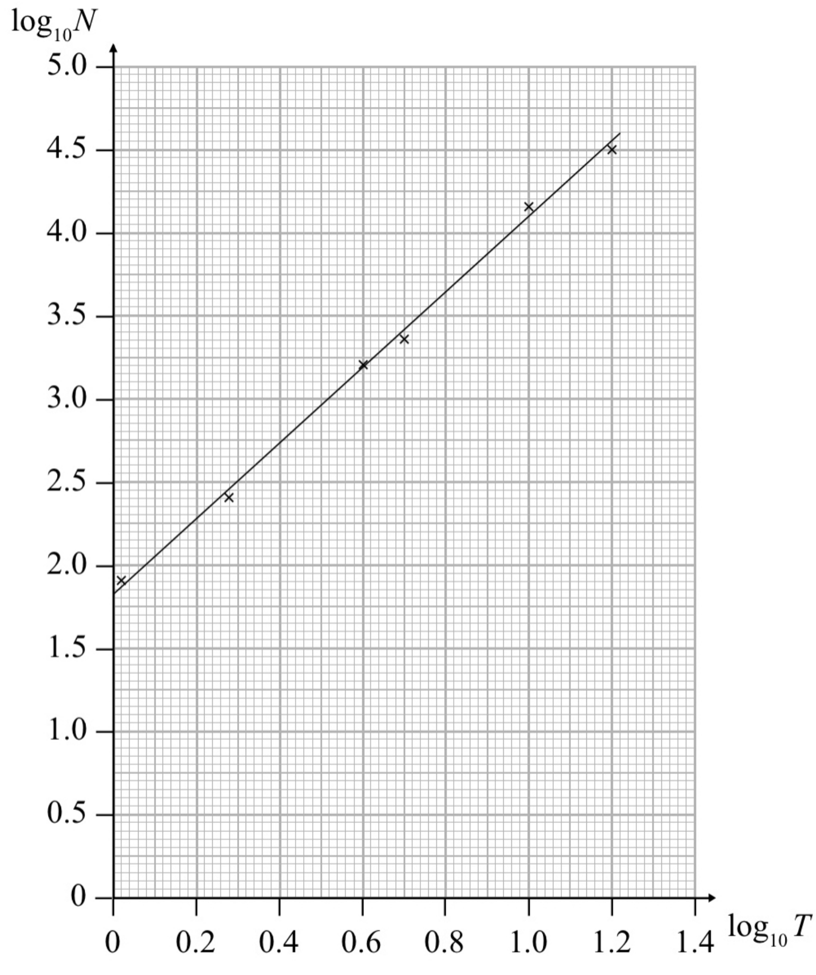


Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

Pearson Edexcel A-Level Mathematics SAMS Paper 1

31st March

15 The height x metres, of a column of water in a fountain display satisfies the differential equation $\frac{dx}{dt} = \frac{8\sin 2t}{3\sqrt{x}}$, where t is the time in seconds after the display begins.

15 (a) Solve the differential equation, given that initially the column of water has zero height.

Express your answer in the form $x = f(t)$

[7 marks]

15 (b) Find the maximum height of the column of water, giving your answer to the nearest cm.

[1 mark]

AQA A-Level Mathematics SAMS Paper 1

April Questions

1st April

1 State the values of $|x|$ for which the binomial expansion of $(3+2x)^{-4}$ is valid.

Circle your answer.

[1 mark]

$|x| < \frac{2}{3}$ $\qquad$ $|x| < 1$ $\qquad$ $|x| < \frac{3}{2}$ $\qquad$ $|x| < 3$

AQA A-Level Mathematics SAMS Paper 2

2nd April

Pearson Edexcel A-Level Mathematics SAMS Paper 2

1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

(3)

3rd April

OCR A (H240) A-Level Mathematics SAMS Paper 2

2 A curve has equation $y = x^5 - 5x^4$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **[3]**

(b) Verify that the curve has a stationary point when $x = 4$. **[2]**

(c) Determine the nature of this stationary point. **[2]**

4th April

3 A curve is defined by the parametric equations

$$x = t^3 + 2, \qquad y = t^2 - 1$$

3 (b) Find a Cartesian equation of the curve.

[2 marks]

AQA A-Level Mathematics SAMS Paper 2

5th April

2. Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos\theta = 2\sin\theta$$

The attempts of two of the students are shown below.

Student *A*	Student *B*
$\cos\theta = 2\sin\theta$	$\cos\theta = 2\sin\theta$
$\tan\theta = 2$	$\cos^2\theta = 4\sin^2\theta$
$\theta = 63.4°$	$1 - \sin^2\theta = 4\sin^2\theta$
	$\sin^2\theta = \frac{1}{5}$
	$\sin\theta = \pm\frac{1}{\sqrt{5}}$
	$\theta = \pm 26.6°$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

Pearson Edexcel A-Level Mathematics SAMS Paper 2

6th April

5 (a) Determine a sequence of transformations which maps the graph of $y = \cos\theta$ onto the graph of $y = 3\cos\theta + 3\sin\theta$

Fully justify your answer.

[6 marks]

5 (b) Hence or otherwise find the least value and greatest value of

$$4 + \left(3\cos\theta + 3\sin\theta\right)^2$$

Fully justify your answer.

[3 marks]

AQA A-Level Mathematics SAMS Paper 2

7th April

6 Helga invests £4000 in a savings account.
After t days, her investment is worth £y.
The rate of increase of y is ky, where k is a constant.

(a) Write down a differential equation in terms of t, y and k. **[1]**

(b) Solve your differential equation to find the value of Helga's investment after t days.
Give your answer in terms of k and t. **[4]**

It is given that $k = \frac{1}{365}\ln\left(1+\frac{r}{100}\right)$ where r % is the rate of interest per annum.
During the first year the rate of interest is 6% per annum.

(c) Find the value of Helga's investment after 90 days. **[2]**

After one year (365 days), the rate of interest drops to 5% per annum.

(d) Find the total time that it will take for Helga's investment to double in value. **[5]**

OCR B (H640) A-Level Mathematics SAMS Paper 1

8th April

4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0,\ x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

Pearson Edexcel A-Level Mathematics SAMS Paper 2

9th April

13 Evaluate $\int_0^1 \frac{1}{1+\sqrt{x}}\,dx$, giving your answer in the form $a + b\ln c$, where a, b and c are integers.

[6]

OCR B(H640) A-Level Mathematics SAMS Paper 1

10th April

7 A student notices that when he adds two consecutive odd numbers together the answer always seems to be the difference between two square numbers.

He claims that this will always be true.

He attempts to prove his claim as follows:

Step 1: Check first few cases

$3+5=8$ and $8=3^2-1^2$

$5+7=12$ and $12=4^2-2^2$

$7+9=16$ and $16=5^2-3^2$

Step 2: Use pattern to predict and check a large example

$101+103=204$

subtract 1 and divide by 2 for the first number

Add 1 and divide by two for the second number

$52^2-50^2=204$ it works!

Step 3: Conclusion

The first few cases work and there is a pattern, which can be used to predict larger numbers.

Therefore, it must be true for all consecutive odd numbers.

7 (a) Explain what is wrong with the student's "proof".

[1 mark]

7 (b) Prove that the student's claim is correct.

[3 marks]

AQA A-Level Mathematics SAMS Paper 2

11th April

10 In the past, the time spent in minutes, by customers in a certain library had mean 32.5 and standard deviation 8.2.

Following a change of layout in the library, the mean time spent in the library by a random sample of 50 customers is found to be 34.5 minutes.

Assuming that the standard deviation remains at 8.2, test at the 5% significance level whether the mean time spent by customers in the library has changed. **[7]**

OCR A (H240) A-Level Mathematics SAMS Paper

12th April

8.

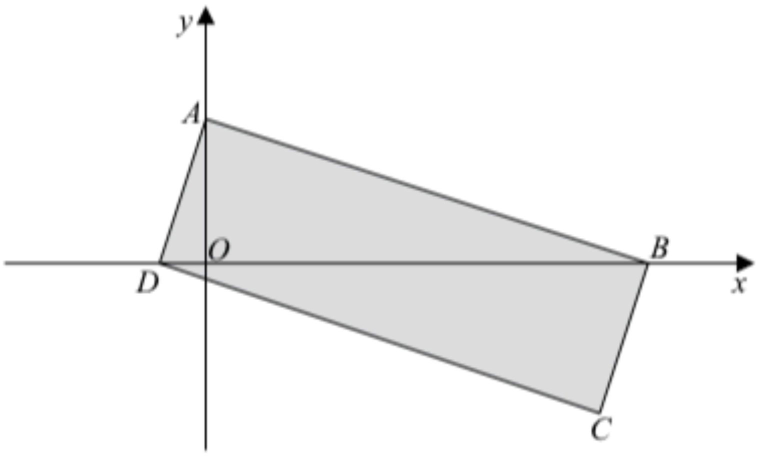


Figure 1

Figure 1 shows a rectangle $ABCD$.

The point A lies on the y-axis and the points B and D lie on the x-axis as shown in Figure 1.

Given that the straight line through the points A and B has equation $5y + 2x = 10$

(a) show that the straight line through the points A and D has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle $ABCD$.

(3)

Pearson Edexcel A-Level Mathematics SAMS Paper 2

13th April

8 A curve has equation $y = 2x\cos 3x + \left(3x^2 - 4\right)\sin 3x$

8 (a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer in the form $\left(mx^2 + n\right)\cos 3x$, where m and n are integers.

[4 marks]

8 (b) Show that the x-coordinates of the points of inflection of the curve satisfy the equation

$$\cot 3x = \frac{9x^2 - 10}{6x}$$

[4 marks]

AQA A-Level Mathematics SAMS Paper 2

14th April

2 A zoologist is investigating the growth of a population of red squirrels in a forest.

She uses the equation $N = \dfrac{200}{1 + 9e^{-\frac{t}{5}}}$ as a model to predict the number of squirrels, N, in the population t weeks after the start of the investigation.

What is the size of the squirrel population at the start of the investigation?

Circle your answer.

[1 mark]

5 20 40 200

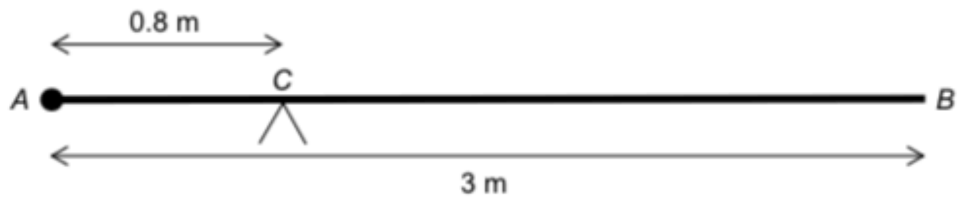
AQA A-Level Mathematics SAMS Paper 2

15th April

11 A uniform rod, *AB*, has length 3 metres and mass 24 kg.

A particle of mass M kg is attached to the rod at *A*.

The rod is balanced in equilibrium on a support at *C*, which is 0.8 metres from *A*.



Find the value of M.

[2 marks]

AQA A-Level Mathematics SAMS Paper 2

16th April

(Temperatures can be converted from Celsius to Fahrenheit using the formula $F = 1.8C + 32$, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

(b) For maximum temperature in October in degrees Fahrenheit, estimate

- the mean
- the standard deviation. **[2]**

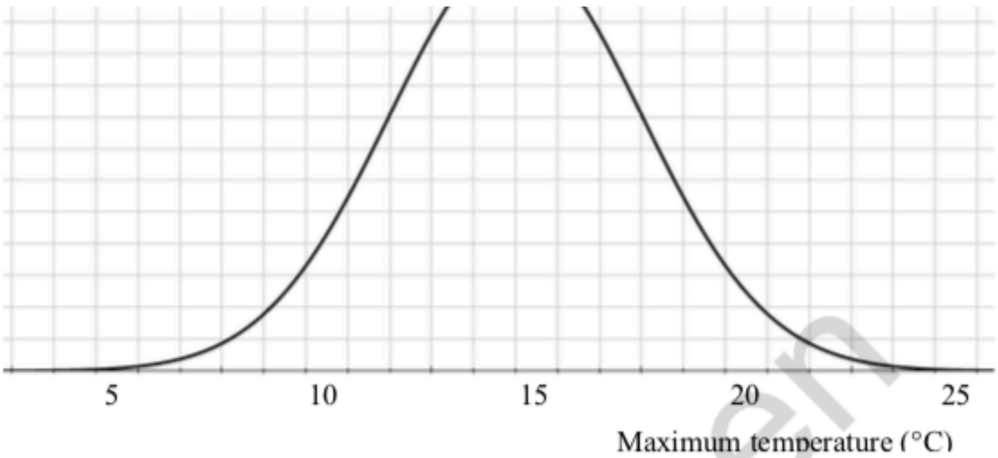


Fig. 6

(a) (i) Use the model to write down the mean of the maximum temperatures. **[1]**

(ii) Explain why the curve indicates that the standard deviation is approximately 3 degrees Celsius. **[1]**

OCR B(H640) A-Level Mathematics SAMS Paper 2

17th April

10. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_\infty = \frac{8}{7} \times S_6$$

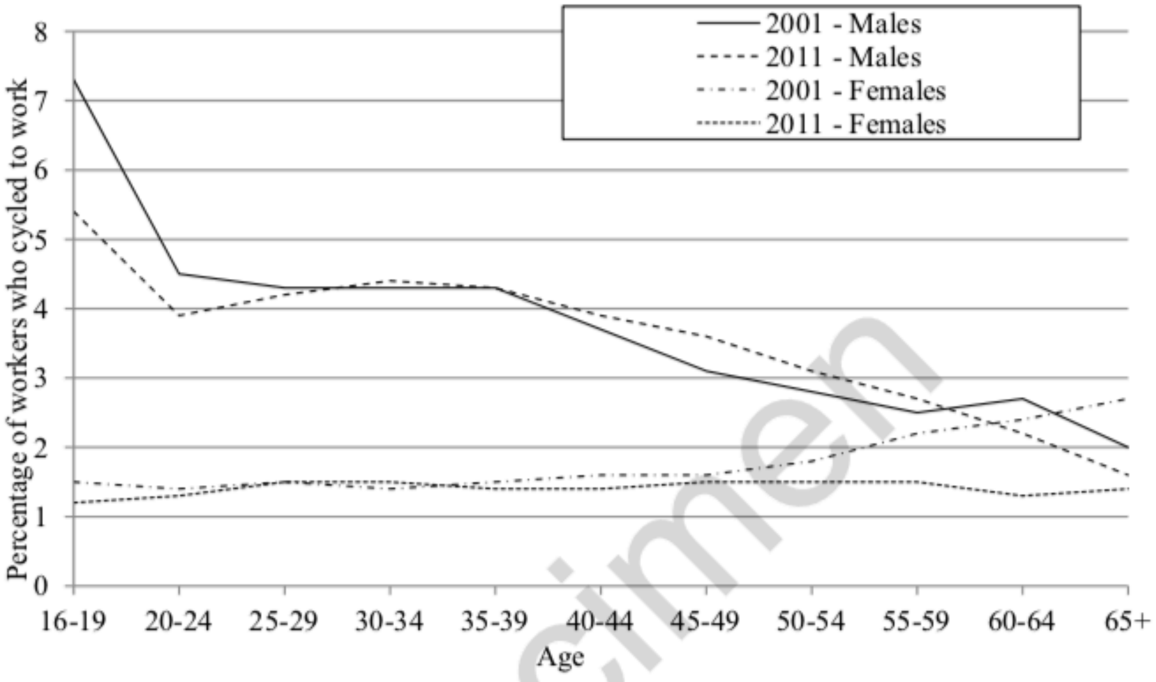
show that $r = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

Pearson Edexcel A-Level Mathematics SAMS Paper 2

18th April

9 The diagram below shows some "Cycle to work" data taken from the 2001 and 2011 UK censuses. The diagram shows the percentages, by age group, of male and female workers in England and Wales, excluding London, who cycled to work in 2001 and 2011.



The following questions refer to the workers represented by the graphs in the diagram.

(a) A researcher is going to take a sample of men and a sample of women and ask them whether or not they cycle to work.

Why would it be more important to stratify the sample of men? **[1]**

A research project followed a randomly chosen large sample of the group of male workers who were aged 30-34 in 2001.

(b) Does the diagram suggest that the proportion of this group who cycled to work has increased or decreased from 2001 to 2011?
Justify your answer. **[2]**

(c) Write down one assumption that you have to make about these workers in order to draw this conclusion. **[1]**

OCR A (H240) A-Level Mathematics SAMS Paper 2

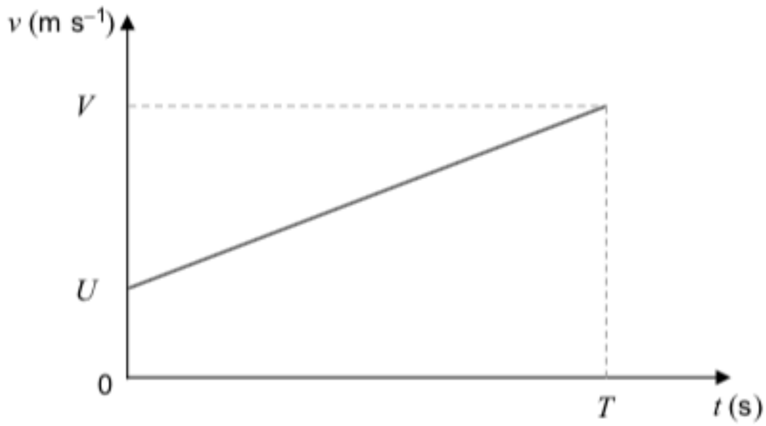
19th April

12 A particle moves on a straight line with a constant acceleration, a m s^{-2}.

The initial velocity of the particle is U m s^{-1}.

After T seconds the particle has velocity V m s^{-1}.

This information is shown on the velocity-time graph.



The displacement, S metres, of the particle from its initial position at time T seconds is given by the formula

$$S = \frac{1}{2}(U + V)T$$

12 (a) By considering the gradient of the graph, or otherwise, write down a formula for a in terms of U, V and T.

[1 mark]

12 (b) Hence show that $V^2 = U^2 + 2aS$

[3 marks]

AQA A-Level Mathematics SAMS Paper 2

20th April

11 Each of the 30 students in a class plays at least one of squash, hockey and tennis.

- 18 students play squash
- 19 students play hockey
- 17 students play tennis
- 8 students play squash and hockey
- 9 students play hockey and tennis
- 11 students play squash and tennis

(a) Find the number of students who play all three sports. **[3]**

A student is picked at random from the class.

(b) Given that this student plays squash, find the probability that this student does not play hockey. **[1]**

Two different students are picked at random from the class, one after the other, without replacement.

(c) Given that the first student plays squash, find the probability that the second student plays hockey. **[4]**

OCR A (H240) A-Level Mathematics SAMS Paper 2

21st April

14. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

Pearson Edexcel A-Level Mathematics SAMS Paper 2

22nd April

12 **Fig. 12** shows the curve $2x^3 + y^3 = 5y$.

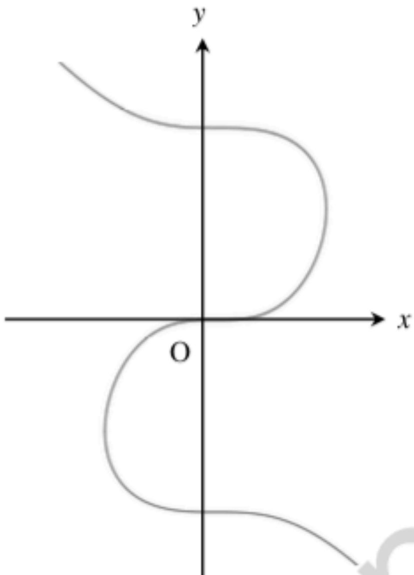


Fig. 12

(a) Find the gradient of the curve $2x^3 + y^3 = 5y$ at the point $(1, 2)$, giving your answer in exact form. **[4]**

(b) Show that all the stationary points of the curve lie on the y-axis. **[2]**

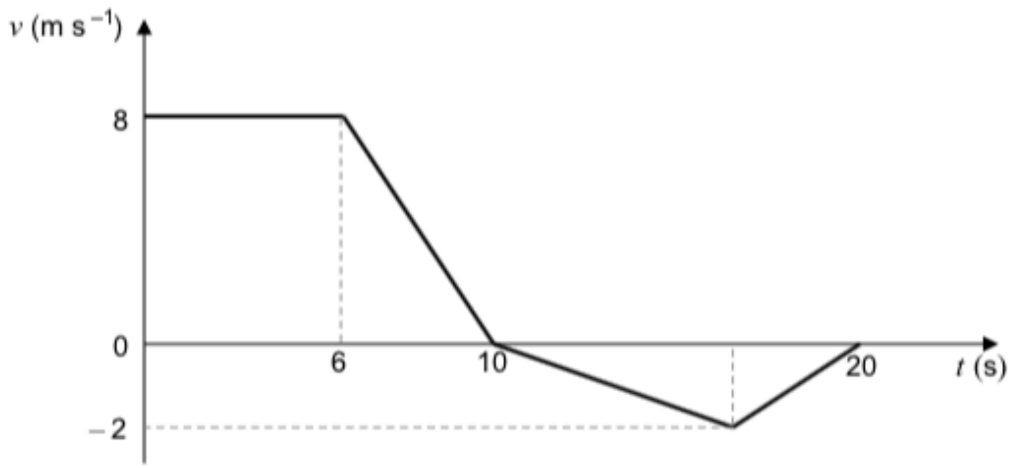
OCR B(H640) A-Level Mathematics SAMS Paper 2

23rd April

14 The graph below models the velocity of a small train as it moves on a straight track for 20 seconds.

The front of the train is at the point A when $t = 0$

The mass of the train is 800kg.



14 (a) Find the total distance travelled in the 20 seconds.

[3 marks]

14 (b) Find the distance of the front of the train from the point A at the end of the 20 seconds.

[1 mark]

14 (c) Find the maximum magnitude of the resultant force acting on the train.

[2 marks]

14 (d) Explain why, in reality, the graph may not be an accurate model of the motion of the train.

[1 mark]

AQA A-Level Mathematics SAMS Paper 2

24th April

11 Suppose x is an irrational number, and y is a rational number, so that $y = \dfrac{m}{n}$,

where m and n are integers and $n \neq 0$.

Prove by contradiction that $x + y$ is not rational. **[4]**

OCR B(H640) A-Level Mathematics SAMS Paper 2

25th April

6. Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0, \quad (a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. **(2)**				
(ii) If $ax > b$ then $x > \frac{b}{a}$ **(2)**				
(iii) The difference between consecutive square numbers is odd. **(2)**				

(Total for Question 6 is 6 marks)

Pearson Edexcel A-Level Mathematics SAMS Paper 2

<u>26th April</u>

15 At time $t = 0$, a parachutist jumps out of an airplane that is travelling horizontally.

The velocity, $\mathbf{v}$ m s^{-1}, of the parachutist at time t seconds is given by:

$$\mathbf{v} = \left(40e^{-0.2t}\right)\mathbf{i} + 50\left(e^{-0.2t} - 1\right)\mathbf{j}$$

The unit vectors **i** and **j** are horizontal and vertical respectively.

Assume that the parachutist is at the origin when $t = 0$

Model the parachutist as a particle.

15 (a) Find an expression for the position vector of the parachutist at time t.

[4 marks]

15 (b) The parachutist opens her parachute when she has travelled 100 metres horizontally.

Find the vertical displacement of the parachutist from the origin when she opens her parachute.

[4 marks]

15 (c) Carefully, explaining the steps that you take, deduce the value of g used in the formulation of this model.

[3 marks]

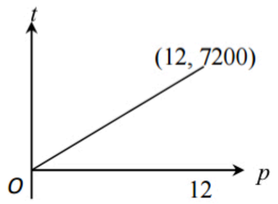
AQA A-Level Mathematics SAMS Paper 2

27th April

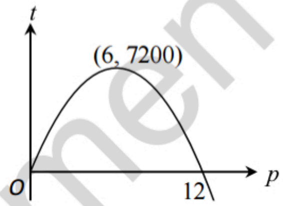
3 A publisher has to choose the price at which to sell a certain new book.
The total profit, £t, that the publisher will make depends on the price, £p.
He decides to use a model that includes the following assumptions.

- If the price is low, many copies will be sold, but the profit on each copy sold will be small, and the total profit will be small.
- If the price is high, the profit on each copy sold will be high, but few copies will be sold, and the total profit will be small.

The graphs below show two possible models.



Model A



Model B

(a) Explain how model A is inconsistent with one of the assumptions given above. **[1]**

(b) Given that the equation of the curve in model B is quadratic, show that this equation is of the form $t = k(12p - p^2)$, and find the value of the constant k. **[2]**

(c) The publisher needs to make a total profit of at least £6400. Use the equation found in part **(b)** to find the range of values within which model B suggests that the price of the book must lie. **[4]**

(d) Comment briefly on how realistic model B may be in the following cases.

- $p = 0$
- $p = 12.1$ **[2]**

OCR A (H240) A-Level Mathematics SAMS Paper 2

28th April

9 A geyser is a hot spring which erupts from time to time. For two geysers, the duration of each eruption, x minutes, and the waiting time until the next eruption, y minutes, are recorded.

(a) For a random sample of 50 eruptions of the first geyser, the correlation coefficient between x and y is 0.758.
The critical value for a 2-tailed hypothesis test for correlation at the 5% level is 0.279.
Explain whether or not there is evidence of correlation in the population of eruptions. **[2]**

The scatter diagram in **Fig. 9** shows the data from a random sample of 50 eruptions of the second geyser.

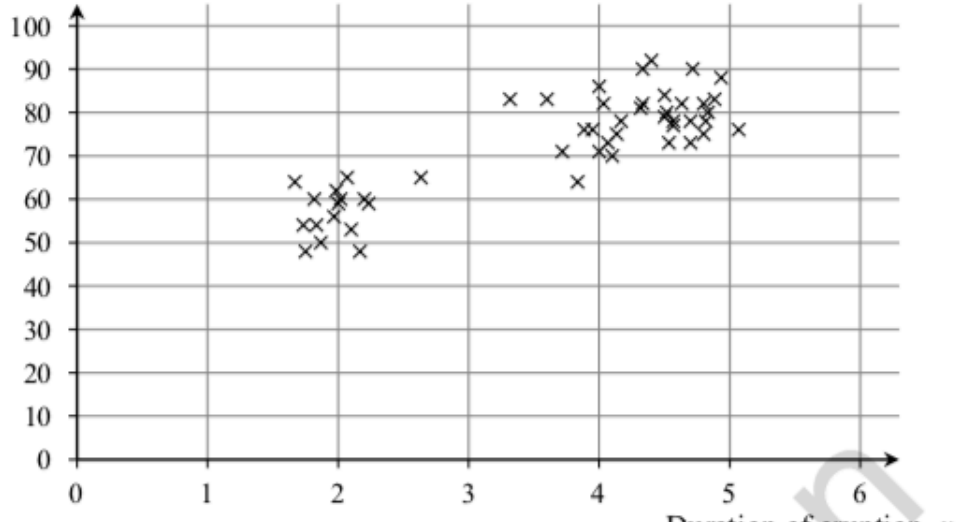


Fig. 9

(b) Stella claims the scatter diagram shows evidence of correlation between duration of eruption and waiting time. Make two comments about Stella's claim. **[2]**

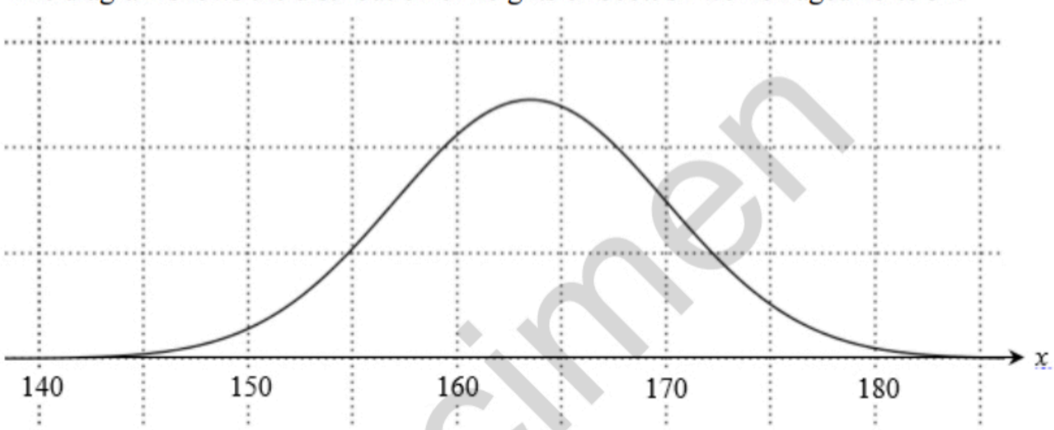
OCR B(H640) A-Level Mathematics SAMS Paper 2

29th April

7 **(a)** The heights of English men aged 25 to 34 are normally distributed with mean 178 cm and standard deviation 8 cm.
Three English men aged 25 to 34 are chosen at random.

Find the probability that all three men have a height less than 194 cm. **[3]**

(b) The diagram shows the distribution of heights of Scottish women aged 25 to 34.



The distribution is approximately normal. Use the diagram in the Printed Answer Booklet to estimate the standard deviation of these heights, explaining your method. **[3]**

OCR A (H240) A-Level Mathematics SAMS Paper 230th April

April 30th

9 (a) Three consecutive terms in an arithmetic sequence are $3e^{-p}, 5, 3e^{p}$

Find the possible values of p. Give your answers in an exact form.

[6 marks]

9 (b) Prove that there is no possible value of q for which $3e^{-q}$, 5, $3e^{q}$ are consecutive terms of a geometric sequence.

[4 marks]

AQA A-Level Mathematics SAMS Paper 2

May Questions

1st May

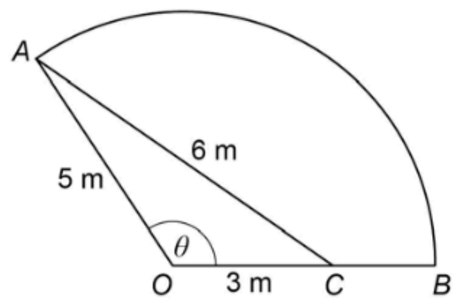
1 **(a)** If $|x| = 3$, find the possible values of $|2x - 1|$. **[3]**

(b) Find the set of values of x for which $|2x - 1| > x + 1$.
Give your answer in set notation. **[4]**

OCR A (H240) A-Level Mathematics SAMS Paper 3

2nd May

2 A wooden frame is to be made to support some garden decking. The frame is to be in the shape of a sector of a circle. The sector *OAB* is shown in the diagram, with a wooden plank *AC* added to the frame for strength. *OA* makes an angle of θ with *OB*.



2 (a) Show that the exact value of $\sin\theta$ is $\dfrac{4\sqrt{14}}{15}$

[3 marks]

2 (b) Write down the value of θ in radians to 3 significant figures.

[1 mark]

2 (c) Find the area of the garden that will be covered by the decking.

[2 marks]

AQA A-Level Mathematics SAMS Paper 3

3rd May

1 Express $\dfrac{2}{x-1} + \dfrac{5}{2x+1}$ as a single fraction. **[2]**

OCR B(H640) A-Level Mathematics SAMS Paper 3

4th May

1. The number of hours of sunshine each day, y, for the month of July at Heathrow are summarised in the table below.

Hours	$0 \leqslant y < 5$	$5 \leqslant y < 8$	$8 \leqslant y < 11$	$11 \leqslant y < 12$	$12 \leqslant y < 14$
Frequency	12	6	8	3	2

A histogram was drawn to represent these data. The $8 \leqslant y < 11$ group was represented by a bar of width 1.5 cm and height 8 cm.

(a) Find the width and the height of the $0 \leqslant y < 5$ group.

(3)

(b) Use your calculator to estimate the mean and the standard deviation of the number of hours of sunshine each day, for the month of July at Heathrow.
Give your answers to 3 significant figures.

(3)

The mean and standard deviation for the number of hours of daily sunshine for the same month in Hurn are 5.98 hours and 4.12 hours respectably.
Thomas believes that the further south you are the more consistent should be the number of hours of daily sunshine.

(c) State, giving a reason, whether or not the calculations in part (b) support Thomas' belief.

(2)

(d) Estimate the number of days in July at Heathrow where the number of hours of sunshine is more than 1 standard deviation above the mean.

(2)

Helen models the number of hours of sunshine each day, for the month of July at Heathrow by $N(6.6, 3.7^2)$.

(e) Use Helen's model to predict the number of days in July at Heathrow when the number of hours of sunshine is more than 1 standard deviation above the mean.

(2)

(f) Use your answers to part (d) and part (e) to comment on the suitability of Helen's model.

(1)

Pearson Edexcel A-Level Mathematics SAMS Paper 3

5th May

14 A survey during 2013 investigated mean expenditure on bread and on alcohol.

The 2013 survey obtained information from 12 144 adults.

The survey revealed that the mean expenditure per adult per week on bread was 127p.

14 (a) For 2012, it is known that the expenditure per adult per week on bread had mean 123p, and a standard deviation of 70p.

14 (a) (i) Carry out a hypothesis test, at the 5% significance level, to investigate whether the mean expenditure per adult per week on bread changed from 2012 to 2013.

Assume that the survey data is a random sample taken from a normal distribution.

[5 marks]

14 (a) (ii) Calculate the greatest and least values for the sample mean expenditure on bread per adult per week for 2013 that would have resulted in acceptance of the null hypothesis for the test you carried out in part **(a)(i)**.

Give your answers to two decimal places.

[2 marks]

14 (b) The 2013 survey revealed that the mean expenditure per adult, per week on alcohol was 324p.

The mean expenditure per adult per week on alcohol for 2009 was 307p.

A test was carried out on the following hypotheses relating to mean expenditure per adult per week on alcohol in 2013.

$H_0 : \mu = 307$

$H_1 : \mu \neq 307$

This test resulted in the null hypothesis, H_0, being rejected.

State, with a reason, whether the test result supports the following statements:

14 (b) (i) the mean UK expenditure on alcohol per adult per week increased by 17p from 2009 to 2013;

[2 marks]

14 (b) (ii) the mean UK consumption of alcohol per adult per week changed from 2009 to 2013.

[2 marks]

AQA A-Level Mathematics SAMS Paper 3

<u>6th May</u>

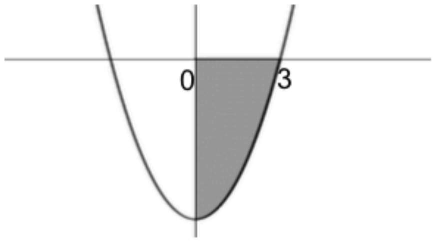
2 **(a)** Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for $\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx$. **[3]**

(b) Explain how the trapezium rule might be used to give a better approximation to the integral given in part **(a)**. **[1]**

OCR A (H240) A-Level Mathematics SAMS Paper 3

7th May

1 The graph of $y = x^2 - 9$ is shown below.



Find the area of the shaded region.
Circle your answer.

[1 mark]

-18 $\quad$ -6 $\quad$ 6 $\quad$ 18

AQA A-Level Mathematics SAMS Paper 3

8th May

2. A meteorologist believes that there is a relationship between the daily mean windspeed, w kn, and the daily mean temperature, t °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

t	13.3	16.2	15.7	16.6	16.3	16.4	19.3	17.1	13.2
w	7	11	8	11	13	8	15	10	11

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained $r = 0.609$

(a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C

(1)

(b) State what is measured by the product moment correlation coefficient.

(1)

(c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero.

(3)

Using the same 9 days a location from the large data set gave $\bar{t} = 27.2$ and $\bar{w} = 3.5$

(d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics.

(1)

Pearson Edexcel A-Level Mathematics SAMS Paper 3

9th May

9 **(a)** Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $0 < \alpha < \frac{1}{2}\pi$ and R is positive and given in exact form. **[4]**

The function $f(\theta)$ is defined by $f(\theta) = \dfrac{1}{(k + \cos\theta + 2\sin\theta)}$, $0 \le \theta \le 2\pi$, k is a constant.

(b) The maximum value of $f(\theta)$ is $\dfrac{(3+\sqrt{5})}{4}$.

Find the value of k. **[3]**

OCR B(H640) A-Level Mathematics SAMS Paper 3

10th May

6 A curve has equation $y = x^2 + kx - 4x^{-1}$ where k is a constant.

Given that the curve has a minimum point when $x = -2$

- find the value of k
- show that the curve has a point of inflection which is not a stationary point. **[7]**

OCR A (H240) A-Level Mathematics SAMS Paper 3

11th May

4 $\int_1^2 x^3 \ln(2x)\,dx$ can be written in the form $p\ln 2 + q$, where p and q are rational numbers.

Find p and q.

[5 marks]

AQA A-Level Mathematics SAMS Paper 3

12th May

6. At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 4$

(6)

Pearson Edexcel A-Level Mathematics SAMS Paper 3

13th May

5 (a) Find the first three terms, in ascending powers of x, in the binomial expansion of $(1+6x)^{\frac{1}{3}}$

[2 marks]

5 (b) Use the result from part **(a)** to obtain an approximation to $\sqrt[3]{1.18}$ giving your answer to 4 decimal places.

[2 marks]

5 (c) Explain why substituting $x = \frac{1}{2}$ into your answer to part **(a)** does not lead to a valid approximation for $\sqrt[3]{4}$.

[1 mark]

AQA A-Level Mathematics SAMS Paper 3

14th May

6 **Fig. 6** shows the curve with equation $y = x^4 - 6x^2 + 4x + 5$.

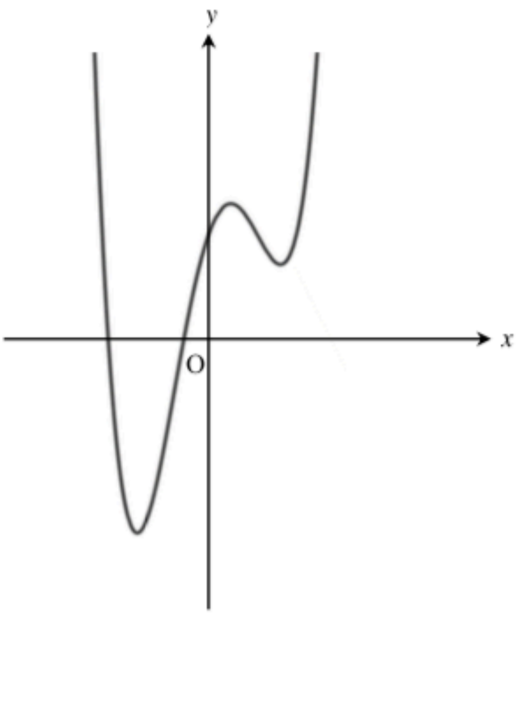


Fig. 6

Find the coordinates of the points of inflection. **[5]**

OCR B(H640) A-Level Mathematics SAMS Paper 3

15th May

7 **(a)** Find $\int 5x^3\sqrt{x^2+1}\,\mathrm{d}x$. **[5]**

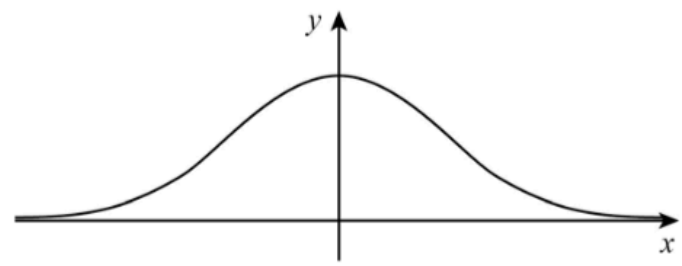
(b) Find $\int \theta \tan^2\theta \,\mathrm{d}\theta$.

You may use the result $\int \tan\theta \,\mathrm{d}\theta = \ln|\sec\theta| + c$. **[5]**

OCR A (H240) A-Level Mathematics SAMS Paper 3

16th May

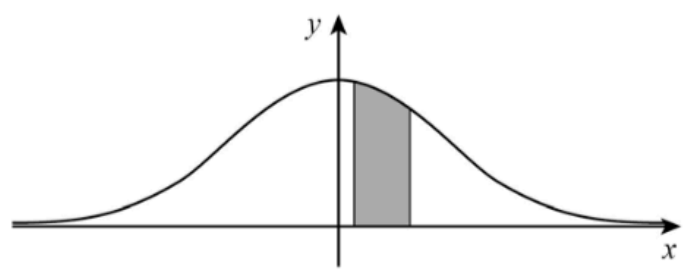
7 The diagram shows part of the graph of $y = \mathrm{e}^{-x^2}$



The graph is formed from two convex sections, where the gradient is increasing, and one concave section, where the gradient is decreasing.

7 (a) Find the values of x for which the graph is concave.

7 (b) The finite region bounded by the x-axis and the lines x = 0.1 and x = 0.5 is shaded.



Use the trapezium rule, with 4 strips, to find an estimate for $\int_{0.1}^{0.5} \mathrm{e}^{-x^2}\,\mathrm{d}x$

Give your estimate to four decimal places.

[3 marks]

7 (c) Explain with reference to your answer in part **(a)**, why the answer you found in part **(b)** is an underestimate.

[2 marks]

7 (d) By considering the area of a rectangle, and using your answer to part **(b)**, prove that the shaded area is 0.4 correct to 1 decimal place.

[3 marks]

AQA A-Level Mathematics SAMS Paper 3

17th May

10 The function $f(x)$ is defined by $f(x) = x^4 + x^3 - 2x^2 - 4x - 2$.

(a) Show that $x = -1$ is a root of $f(x) = 0$. **[1]**

(b) Show that another root of $f(x) = 0$ lies between $x = 1$ and $x = 2$. **[2]**

(c) Show that $f(x) = (x+1)g(x)$, where $g(x) = x^3 + ax + b$ and a and b are integers to be determined. **[3]**

(d) Without further calculation, explain why $g(x) = 0$ has a root between $x = 1$ and $x = 2$. **[1]**

(e) Use the Newton-Raphson formula to show that an iteration formula for finding roots of $g(x) = 0$ may be written

$$x_{n+1} = \frac{2x_n^{\ 3} + 2}{3x_n^{\ 2} - 2}.$$

Determine the root of $g(x) = 0$ which lies between $x = 1$ and $x = 2$ correct to 4 significant figures. **[3]**

OCR B(H640) A-Level Mathematics SAMS Paper 3

18th May

2 Find the first four terms of the binomial expansion of $(1-2x)^{\frac{1}{2}}$.

State the set of values of x for which the expansion is valid. **[4]**

OCR B(H640) A-Level Mathematics SAMS Paper 3

19th May

4 For a small angle θ, where θ is in radians, show that $1+\cos\theta-3\cos^2\theta \approx -1+\frac{5}{2}\theta^2$. **[4]**

OCR A (H240) A-Level Mathematics SAMS Paper 3

20th May

7. A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \dfrac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\dfrac{4}{5}g$.

(a) Find the value of μ.

(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

(2)

Pearson Edexcel A-Level Mathematics SAMS Paper 3

21st May

6 Find the value of $\displaystyle\int_1^2 \frac{6x+1}{6x^2-7x+2}\,\mathrm{d}x$, expressing your answer in the form

$m\ln 2 + n\ln 3$, where m and n are integers.

[8 marks]

AQA A-Level Mathematics SAMS Paper 3

22nd May

3 Show that points A (1, 4, 9), B (0, 11, 17) and C (3, –10, –7) are collinear. **[4]**

OCR B(H640) A-Level Mathematics SAMS Paper 3

23rd May

15.

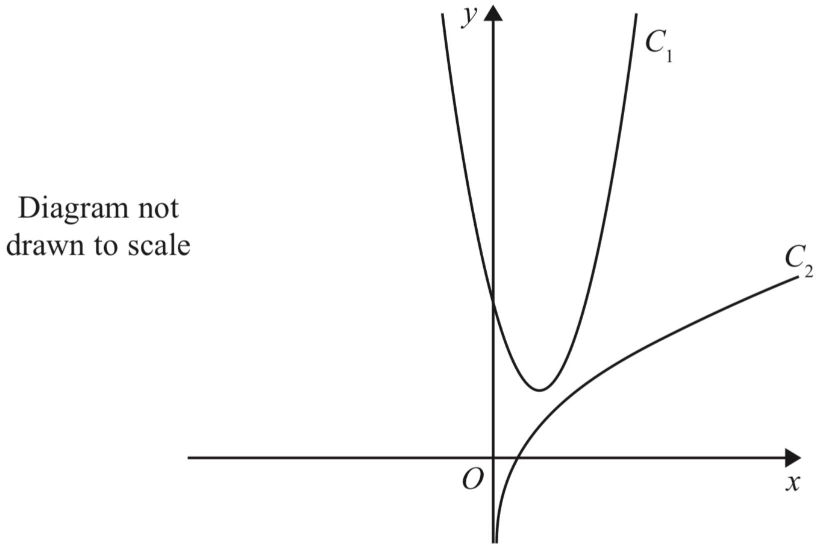


Figure 3

The curve C_1, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1

The curve C_2, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$.

The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(8)

Pearson Edexcel AS-Level Mathematics SAMS Paper 2

21st May

15 A sample of 200 households was obtained from a small town.

Each household was asked to complete a questionnaire about their purchases of takeaway food.

A is the event that a household regularly purchases Indian takeaway food.

B is the event that a household regularly purchases Chinese takeaway food.

It was observed that $\mathrm{P}(B \mid A) = 0.25$ and $\mathrm{P}(A \mid B) = 0.1$

Of these households, 122 indicated that they did **not** regularly purchase Indian or Chinese takeaway food.

A household is selected at random from those in the sample.

Find the probability that the household regularly purchases **both** Indian and Chinese takeaway food.

[6 marks]

AQA A-Level Mathematics SAMS Paper 3

24th May

11 The curve $y = \mathrm{f}(x)$ is defined by the function $\mathrm{f}(x) = \mathrm{e}^{-x}\sin x$ with domain $0 \le x \le 4\pi$.

(a) **(i)** Show that the x-coordinates of the stationary points of the curve $y = \mathrm{f}(x)$, when arranged in increasing order, form an arithmetic sequence.

(ii) Show that the corresponding y-coordinates form a geometric sequence. **[9]**

(b) Would the result still hold with a larger domain? Give reasons for your answer. **[1]**

OCR B(H640) A-Level Mathematics SAMS Paper 3

25th May

3 **In this question you must show detailed reasoning.**

Given that $5\sin 2x = 3\cos x$, where $0^\circ < x < 90^\circ$, find the exact value of $\sin x$. **[4]**

OCR A (H240) A-Level Mathematics SAMS Paper 3

26th May

8 Edna wishes to investigate the energy intake from eating out at restaurants for the households in her village.

She wants a sample of 100 households.

She has a list of all 2065 households in the village.

Ralph suggests this selection method.

> "Number the households 0000 to 2064. Obtain 100 different four-digit random numbers between 0000 and 2064 and select the corresponding households for inclusion in the investigation."

8 (a) What is the population for this investigation?

Circle your answer.

[1 mark]

Edna and Ralph	The 2065 households in the village	The energy intake for the village from eating out	The 100 households selected

8 (b) What is the sampling method suggested by Ralph?

Circle your answer.

[1 mark]

Opportunity	Random number	Continuous random variable	Simple random

AQA A-Level Mathematics SAMS Paper 3

10.

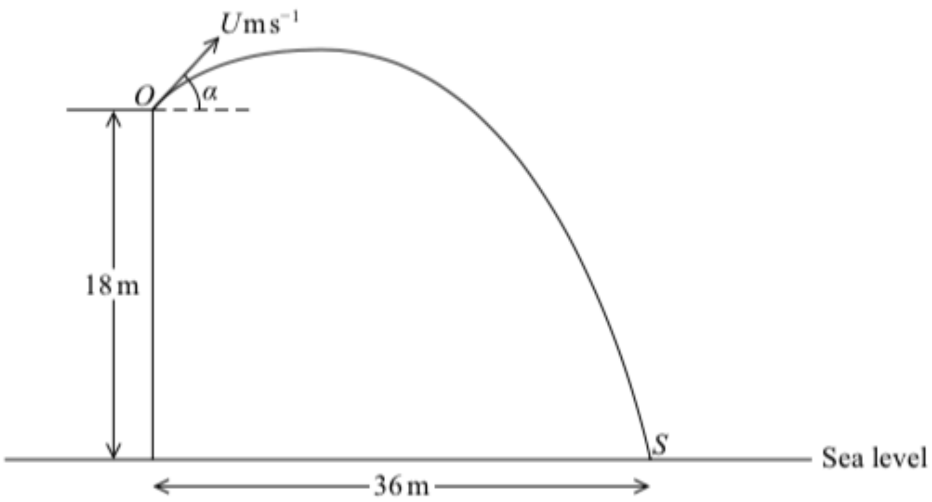


Figure 2

A boy throws a stone with speed $U\,\text{m s}^{-1}$ from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10\,\text{m s}^{-2}$

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(5)

(c) Suggest two improvements that could be made to the model.

(2)

27th May

Pearson Edexcel A-Level Mathematics SAMS Paper 3

28th May

10 A body of mass 20 kg is on a rough plane inclined at angle α to the horizontal.
The body is held at rest on the plane by the action of a force of magnitude P N.
The force is acting up the plane in a direction parallel to a line of greatest slope of the plane.
The coefficient of friction between the body and the plane is μ.

(a) When $P = 100$, the body is on the point of sliding down the plane.

Show that $g\sin\alpha = g\mu\cos\alpha + 5$. **[4]**

(b) When P is increased to 150, the body is on the point of sliding up the plane.

Use this, and your answer to part **(a)**, to find an expression for α in terms of g. **[3]**

OCR A (H240) A-Level Mathematics SAMS Paper 3

29th May

8 In **Fig. 8**, OAB is a thin bent rod, with OA = 1 m, AB = 2 m and angle OAB = 120°.
Angles θ, ϕ and h are as shown in **Fig. 8**.

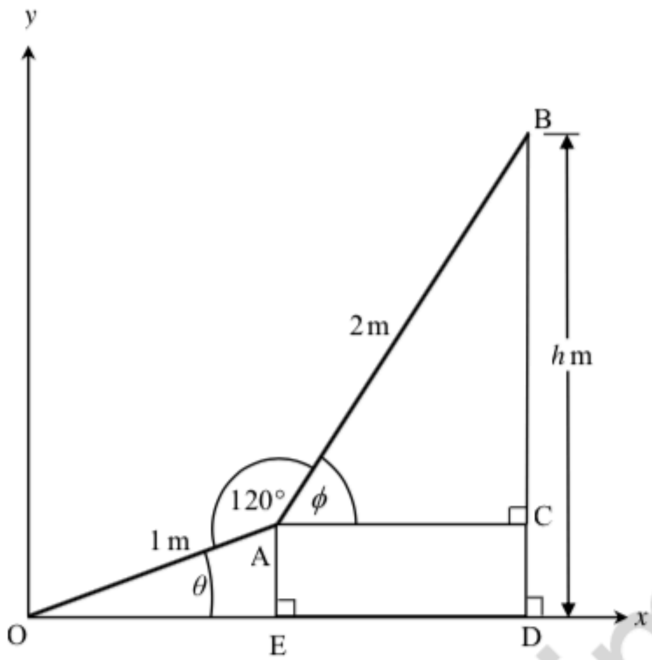


Fig. 8

(a) Show that $h = \sin\theta + 2\sin(\theta + 60°)$. **[3]**

The rod is free to rotate about the origin so that θ and ϕ vary. You may assume that the result for h in part **(a)** holds for all values of θ.

(b) Find an angle θ for which $h = 0$. **[5]**

OCR B(H640) A-Level Mathematics SAMS Paper 3

30th May

11 In this question the unit vectors **i** and **j** are in the directions east and north respectively.

A particle of mass 0.12 kg is moving so that its position vector **r** metres at time t seconds is given by $\mathbf{r} = 2t^3\mathbf{i} + \left(5t^2 - 4t\right)\mathbf{j}$.

(a) Show that when $t = 0.7$ the bearing on which the particle is moving is approximately 044°. **[3]**

(b) Find the magnitude of the resultant force acting on the particle at the instant when $t = 0.7$. **[4]**

(c) Determine the times at which the particle is moving on a bearing of 045°. **[2]**

OCR A (H240) A-Level Mathematics SAMS Paper 3

31st May

9 A survey has found that, of the 2400 households in Growmore, 1680 eat home-grown fruit and vegetables.

9 (a) Using the binomial distribution, find the probability that, out of a random sample of 25 households in Growmore, exactly 22 eat home-grown fruit and vegetables.

[2 marks]

9 (b) Give a reason why you would **not** expect your calculation in part **(a)** to be valid for the 25 households in Gifford Terrace, a residential road in Growmore.

[1 mark]

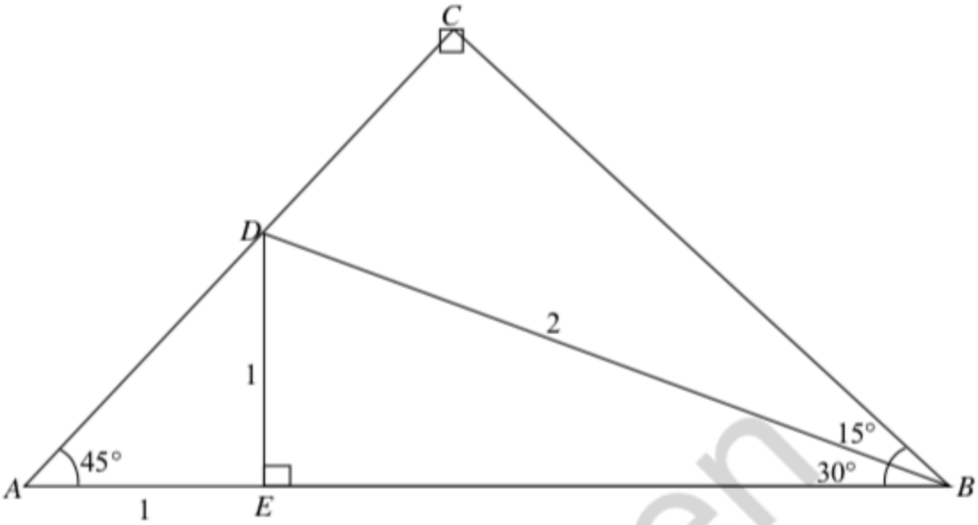
AQA A-Level Mathematics SAMS Paper 3

June Questions

1st June

8 In this question you must show detailed reasoning.

The diagram shows triangle ABC.



The angles CAB and ABC are each 45°, and angle $ACB = 90^\circ$.
The points D and E lie on AC and AB respectively. $AE = DE = 1$, $DB = 2$.
Angle $BED = 90^\circ$, angle $EBD = 30^\circ$ and angle $DBC = 15^\circ$.

(a) Show that $BC = \dfrac{\sqrt{2}+\sqrt{6}}{2}$. **[3]**

(b) By considering triangle BCD, show that $\sin 15^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$. **[3]**

OCR A (H240) A-Level Mathematics SAMS Paper 3

2nd June

3 A circular ornamental garden pond, of radius 2 metres, has weed starting to grow and cover its surface.

As the weed grows, it covers an area of A square metres. A simple model assumes that the weed grows so that the rate of increase of its area is proportional to A.

3 (a) Show that the area covered by the weed can be modelled by

$$A = Be^{kt}$$

where B and k are constants and t is time in days since the weed was first noticed.

[4 marks]

3 (b) When it was first noticed, the weed covered an area of 0.25 m^2. Twenty days later the weed covered an area of 0.5 m^2

3 (b) (i) State the value of B.

[1 mark]

3 (b) (iii) How many days does it take for the weed to cover half of the surface of the pond?

[2 marks]

3 (b) (ii) Show that the model for the area covered by the weed can be written as

$$A = 2^{\frac{t}{20} - 2}$$

3 (c) State one limitation of the model.

[1 mark]

3 (d) Suggest one refinement that could be made to improve the model.

[1 mark]

AQA A-Level Mathematics SAMS Paper 3

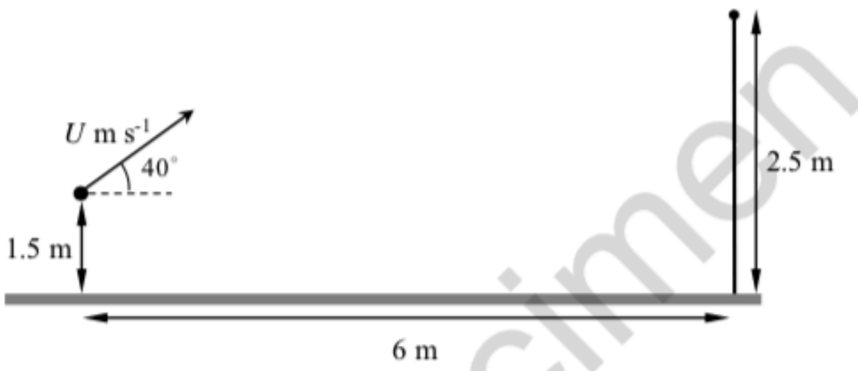
<u>3rd June</u>

12 A girl is practising netball.
She throws the ball from a height of 1.5 m above horizontal ground and aims to get the ball through a hoop.
The hoop is 2.5 m vertically above the ground and is 6 m horizontally from the point of projection.

The situation is modelled as follows.

- The initial velocity of the ball has magnitude U m s^{-1}.
- The angle of projection is 40°.
- The ball is modelled as a particle.
- The hoop is modelled as a point.

This is shown on the diagram below.



(a) For $U = 10$, find

(i) the greatest height above the ground reached by the ball **[5]**

(ii) the distance between the ball and the hoop when the ball is vertically above the hoop.

[4]

(b) Calculate the value of U which allows her to hit the hoop. **[3]**

(c) How appropriate is this model for predicting the path of the ball when it is thrown by the girl? **[1]**

(d) Suggest one improvement that might be made to this model. **[1]**

OCR A (H240) A-Level Mathematics SAMS Paper 3

4th June

4. Given that

$$P(A) = 0.35 \qquad P(B) = 0.45 \qquad \text{and} \qquad P(A \cap B) = 0.13$$

find

(a) $P(A' \mid B')$

(2)

(b) Explain why the events A and B are not independent.

(1)

The event C has $P(C) = 0.20$

The events A and C are mutually exclusive and the events B and C are statistically independent

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region.

(5)

(d) Find $P([B \cup C]')$

(2)

Pearson Edexcel A-Level Mathematics SAMS Paper 3

5th June

7 By finding a counter example, disprove the following statement.

If p and q are non-zero real numbers with $p < q$, then $\frac{1}{p} > \frac{1}{q}$. **[2]**

OCR B(H640) A-Level Mathematics SAMS Paper 3

6th June

10 Some information from the Large Data Set is given in Figures 1 and 2 below.

Figure 1

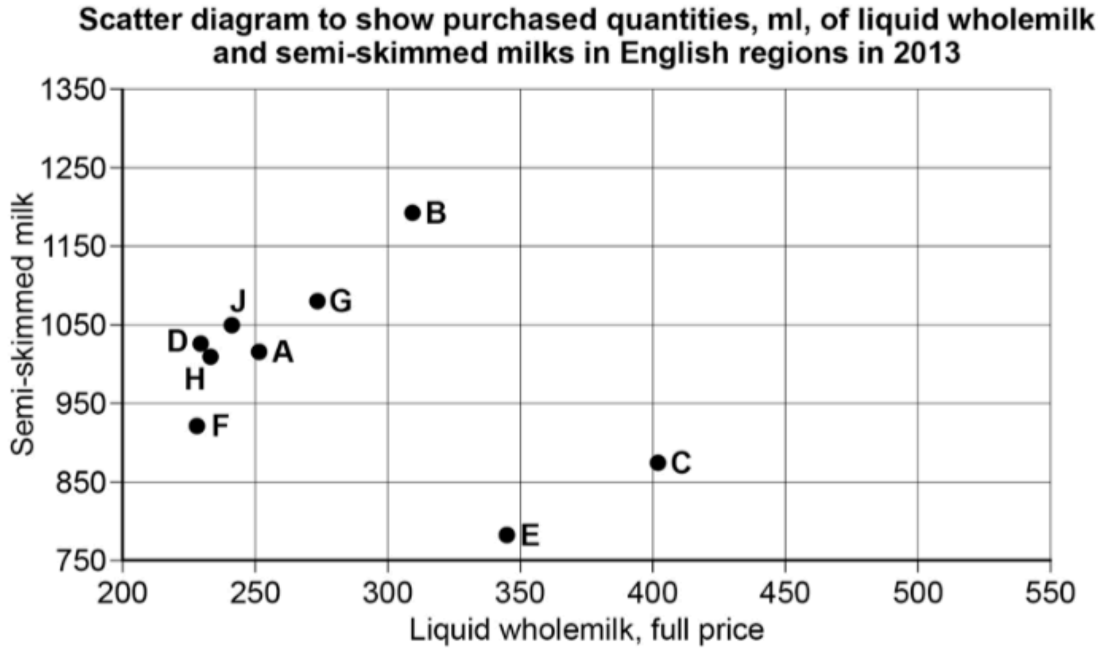
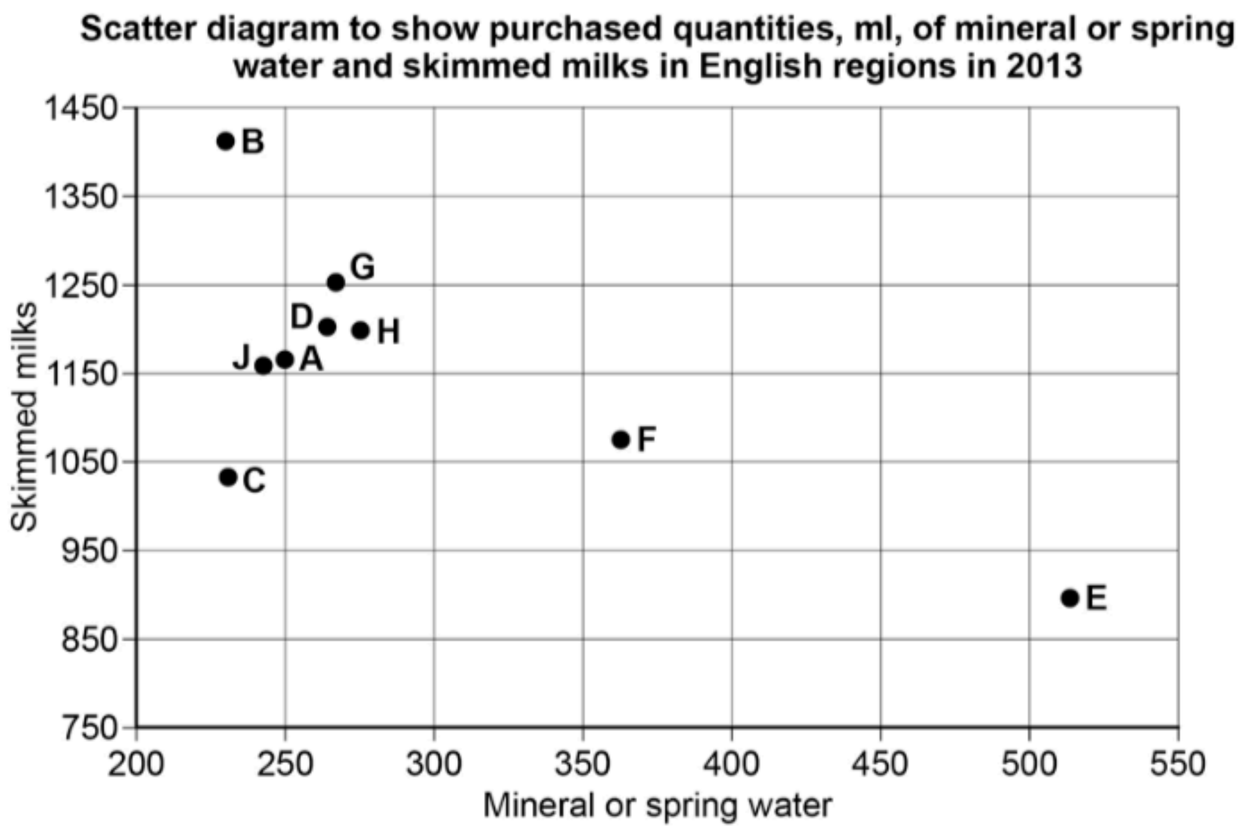


Figure 2



10 (a) Give a reason why the recorded vertical data values are higher for each region in Figure 2 than in Figure 1

[1 mark]

10 (b) (i) Describe the correlation between 'Semi-skimmed milk' and 'Liquid wholemilk, full price'.

[2 marks]

10 (b) (ii) Bilal claims that Figure 2 indicates that when people drink more mineral or spring water they tend to drink less skimmed milk.

Comment on Bilal's claim.

10 (c) Suggest, with a reason, which region is indicated by the letter E. Use your knowledge of the Large Data Set to support your answer.

[2 marks]

AQA A-Level Mathematics SAMS Paper 3

7th June

13 Particle A, of mass m kg, lies on the plane Π_1 inclined at an angle of $\tan^{-1}\frac{3}{4}$ to the horizontal.

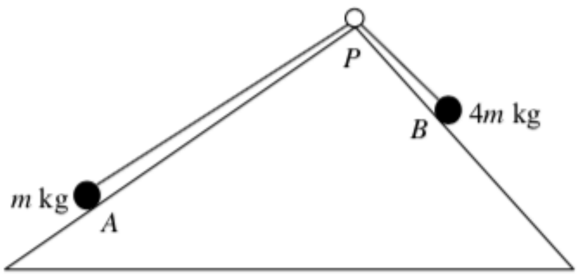
Particle B, of $4m$ kg, lies on the plane Π_2 inclined at an angle of $\tan^{-1}\frac{4}{3}$ to the horizontal.

The particles are attached to the ends of a light inextensible string which passes over a smooth pulley at P.

The coefficient of friction between particle A and Π_1 is $\frac{1}{3}$ and plane Π_2 is smooth.

Particle A is initially held at rest such that the string is taut and lies in a line of greatest slope of each plane.

This is shown on the diagram below.



(a) Show that when A is released it accelerates towards the pulley at $\frac{7g}{15}$ m s^{-2}. **[6]**

(b) Assuming that A does not reach the pulley, show that it has moved a distance of $\frac{1}{4}$ m when its speed is $\sqrt{\frac{7g}{30}}$ m s^{-1}. **[2]**

OCR A (H240) A-Level Mathematics SAMS Paper 3

8th June

11 Terence owns a local shop. His shop has three checkouts, at least one of which is always staffed.

A regular customer observed that the probability distribution for N, the number of checkouts that are staffed at any given time during the spring, is

$$\mathrm{P}(N = n) = \begin{cases} \dfrac{3}{4}\left(\dfrac{1}{4}\right)^{n-1} & \text{for } n = 1, 2 \\ k & \text{for } n = 3 \end{cases}$$

11 (a) Find the value of k.

[1 mark]

11 (b) Find the probability that a customer, visiting Terence's shop during the spring, will find at least 2 checkouts staffed.

[2 marks]

AQA A-Level Mathematics SAMS Paper 3

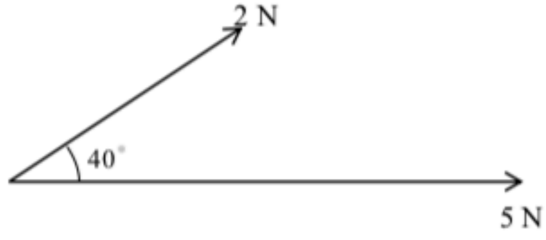
9th June

4 Show that $\sum_{r=1}^{4} \ln \frac{r}{r+1} = -\ln 5$. **[3]**

OCR B(H640) A-Level Mathematics SAMS Paper 3

10th June

9 Two forces, of magnitudes 2 N and 5 N, act on a particle in the directions shown in the diagram below.



(a) Calculate the magnitude of the resultant force on the particle. **[3]**

(b) Calculate the angle between this resultant force and the force of magnitude 5 N. **[1]**

OCR A (H240) A-Level Mathematics SAMS Paper 3

11th June

12 During the 2006 Christmas holiday, John, a maths teacher, realised that he had fallen ill during 65% of the Christmas holidays since he had started teaching.

In January 2007, he increased his weekly exercise to try to improve his health.

For the next 7 years, he only fell ill during 2 Christmas holidays.

12 (a) Using a binomial distribution, investigate, at the 5% level of significance, whether there is evidence that John's rate of illness during the Christmas holidays had decreased since increasing his weekly exercise.

[6 marks]

12 (b) State **two** assumptions, regarding illness during the Christmas holidays, that are necessary for the distribution you have used in part **(a)** to be valid.

For **each** assumption, comment, in context, on whether it is likely to be correct.

[4 marks]

AQA A-Level Mathematics SAMS Paper 3

12th June

5. A company sells seeds and claims that 55% of its pea seeds germinate.

(a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.

(1)

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

(b) Assuming that the company's claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate.

(3)

(c) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.

(1)

A random sample of 240 pea seeds was planted and 150 of these seeds germinated.

(d) Assuming that the company's claim is correct, use a normal approximation to find the probability that at least 150 pea seeds germinate.

(3)

(e) Using your answer to part (d), comment on whether or not the proportion of the company's pea seeds that germinate is different from the company's claim of 55%

(1)

Pearson Edexcel A-Level Mathematics SAMS Paper 3

13th June

13 In the South West region of England, 100 households were randomly selected and, for each household, the weekly expenditure, £X, per person on food and drink was recorded.

The maximum amount recorded was £40.48 and the minimum amount recorded was £22.00

The results are summarised below, where $\bar{x}$ denotes the sample mean.

$$\sum x = 3046.14 \qquad \sum (x - \bar{x})^2 = 1746.29$$

13 (a) (i) Find the mean of X

Find the standard deviation of X

[2 marks]

13 (a) (ii) Using your results from part **(a)(i)** and other information given, explain why the normal distribution can be used to model X.

[2 marks]

13 (a) (iii) Find the probability that a household in the South West spends less than £25.00 on food and drink per person per week.

[1 mark]

13 (b) For households in the North West of England, the weekly expenditure, £Y, per person on food and drink can be modelled by a normal distribution with mean £29.55

It is known that $\mathrm{P}(Y < 30) = 0.55$

Find the standard deviation of Y, giving your answer to one decimal place.

[3 marks]

AQA A-Level Mathematics SAMS Paper 3