FP3 'Keeping Time'

1) Solve the equation $7\operatorname{sech} x - \tanh x = 5$. Give your answers in the form $\ln a$, where a is a rational number.
2) For the vectors $\boldsymbol{a} = 2\boldsymbol{i} + \boldsymbol{j}, \boldsymbol{b} = \boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}\,, \boldsymbol{c} = 3\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$ find
   a) $\boldsymbol{b} \times \boldsymbol{c}$
   b) $\boldsymbol{a}\,.\,(\boldsymbol{b} \times \boldsymbol{c})$
3) Find the exact value of

$$\int_{-2}^{1} \frac{1}{x^2 + 4x + 13}\, dx$$

4) Show that the line with equation $\boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k} + \lambda(2\boldsymbol{i} + \boldsymbol{j})$ where λ is a scalar parameter lies in the plane with equation $\boldsymbol{r}.\,(\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}) = -1$.
5) Use the identity $\sec^2 A = 1 + \tan^2 A$ to find a reduction formula for

$$I_n = \int_0^{\frac{\pi}{4}} \tan^{\mathrm{n}} x \;dx$$

6) Show that $\frac{d}{dx}(\operatorname{arsinh} x) = \frac{1}{\sqrt{x^2+1}}$
7) The hyperbola H has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

   Find
   a) The coordinates of the foci of H.
   b) The equations of the directrices of H.
8) It is given that $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ is an eigenvector of the matrix A, where

$$A = \begin{pmatrix} 4 & 2 & 3 \\ 2 & b & 0 \\ a & 1 & 8 \end{pmatrix}$$

   And a and b are constants.
   a) Find the eigenvalue of A corresponding to the eigenvector $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$
   b) Find the values of a and b.
9) Find the area of the surface generated as the arc of the curve with equation $y = \cosh x$, between the point $(0,1)$ and $\left(\ln 2\,, \frac{5}{4}\right)$ is rotated completely about the y –axis.
10) The plane P has equation

$$\boldsymbol{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + \mu\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$$

   a) Find a vector perpendicular to the plane P.

b) The line l passes through the point $A(1,3,3)$and meets P at (3,1,2). Find the acute angle between the plane P and the line l to the nearest degree.

11) Using the definitions of hyperbolic functions in terms of exponentials

a) Show that $\operatorname{sech}^2 x \ = 1 - \tanh^2 x$

b) Solve the equation $4 \sinh x - 3 \cosh x = 3$

12) The point P lies on the ellipse E with equation

$$\frac{x^2}{36} + \frac{y^2}{9} = 1$$

The foot of the perpendicular from the point P to the line $x = 8$ is labelled N. The midpoint of PN is denoted M.

a) Sketch the graph of the ellipse E, showing also the line $x = 8$ and a possible position for the line PN.

b) Find an equation of the locus of M as P moves around the ellipse.

c) Show that this locus is a circle and state its centre and radius.