FP2 'Keeping Time'

1)

a) Express $\frac{3}{(3r-1)(3r+2)}$ in partial fractions.

b) Hence, show that

$$\sum_{r=1}^{n} \frac{3}{(3r-1)(3r+2)} = \frac{3n}{2(3n+2)}$$

c) Evaluate $\sum_{r=100}^{1000} \frac{3}{(3r-1)(3r+2)}$ giving your answer to 3 significant figures.

2) Find the area enclosed by the cardioid with polar equation $r = 2a(1 + \cos\theta)$

3) Find the general solution of the differential equation

$$\frac{dy}{dx} + \left(\frac{1}{x}\right) y = x^2$$

4) Find the values of x for which

$$\frac{2x}{x-1} > x$$

5) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 5y = 3\cos(5x)$$

6) Find the series expansion for $y = \sqrt{8 + e^x}$ in ascending powers of x, up to and including the term in x^2.

7) A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + 2i}{iz}, \qquad z \neq 0$$

The transformation maps points on the real axis in the z-plane onto a line in the w-plane. Find the equation of this line.

8) Solve the equation $z^4 = -2 + (2\sqrt{3})i$ giving the roots in the form $r(\cos\theta + i\sin\theta)$, where $-\pi < \theta \leq \pi$.

9) Consider the differential equation

$$\frac{d^2y}{dx^2} = xy + \frac{dy}{dx}$$

Given that $\frac{dy}{dx} = 2$ and $y = 1$ at $x = 1$, find the values of $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ at $x = 1$. Hence find the solution of the differential equation as a series in ascending powers of $(x-1)$, up to and including the term in $(x-1)^3$.

10) Express $\cos 5\theta$ in terms of $\cos\theta$.

11) Sketch the locus of z such that $|z - 3| = 2|z - 1 + i|$

12) Show that the transformation $z = y^{\frac{1}{2}}$ transforms the differential equation

$$\frac{dy}{dx} - 4y\tan x = 2y^{\frac{1}{2}}$$

Into the differential equation

$$\frac{dz}{dx} - 2z \tan x = 1$$

Hence solve the original differential equation.