FP2 June 2012 – Harder Version

1) Find the set of values for which $|x^2 - 4| > 3x$

2) A curve C has polar equation $r = 1 + 2\cos\theta$, $0 \leq \theta \leq \frac{\pi}{2}$. At the point P on C, the tangent to C is parallel to the initial line. Given that O is the pole, find the exact length of the line OP.

3) Solve the equation $z^4 = -2 + \left(2\sqrt{3}\right)i$ giving your answers in modulus-argument form.

4) Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2\cos t - \sin t$$

5) Given that $y = 1$ at $x = 1$ find a series solution for y in ascending powers of $(x - 1)$ up to and including the term in $(x - 1)^3$ where y is the solution of the differential equation below.

$$x\frac{dy}{dx} = 3x + y^2$$

6) Show, using the method of differences that

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{n(an+b)}{4(n+1)(n+2)}$$

where a and b are constants to be found. Hence find an expression for $\sum_{r=n+1}^{2n} \frac{1}{r(r+2)}$

7) Using the substitution $y = vx$ find the general solution of the differential equation below in the form $y = f(x)$. Given that $y = 2$ at $x = 1$ find also the value of $\frac{dy}{dx}$ at $x = 1$.

$$3xy^2\frac{dy}{dx} = x^3 + y^3$$

8) Find the locus of the complex number z such that $|z - 6i| = 2|z - 3|$. Find also the complex number for which both $|z - 6i| = 2|z - 3|$ and $arg(z - 6) = -\frac{3\pi}{4}$ illustrating your answer on a diagram.