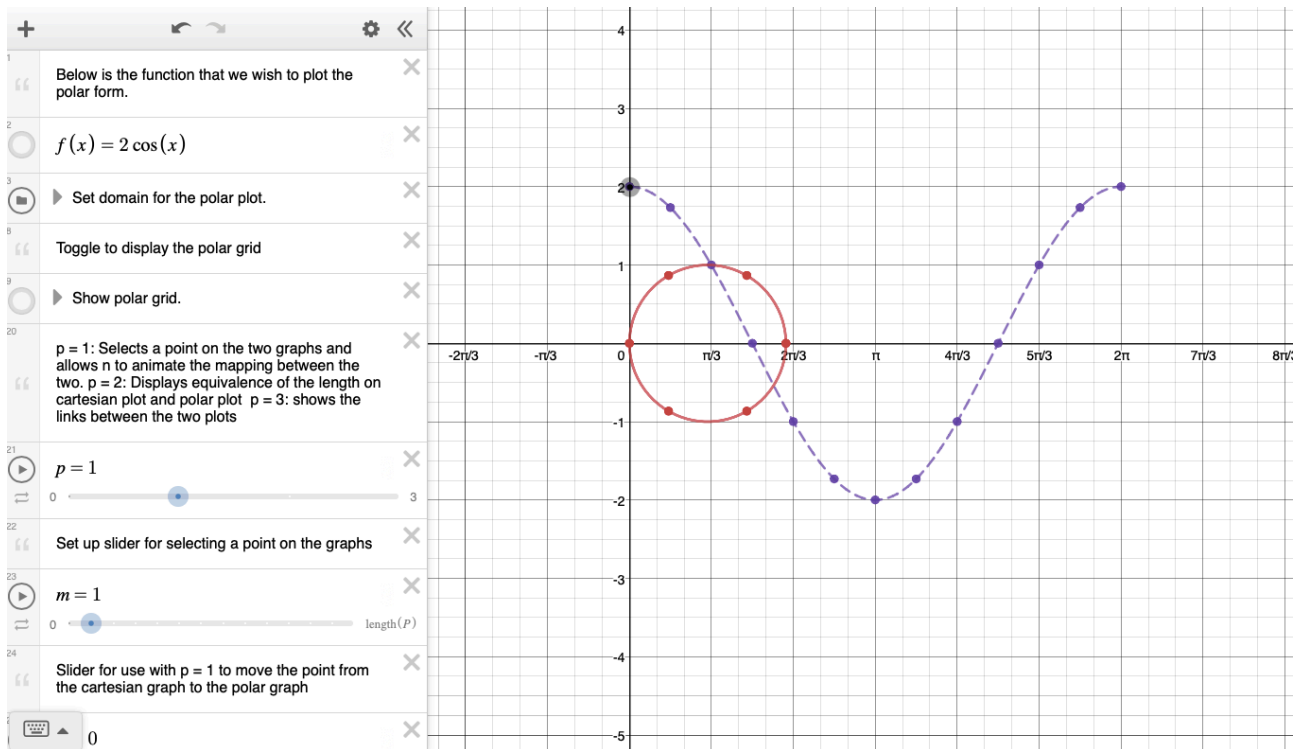


Cartesian to Polar Plotting Desmos

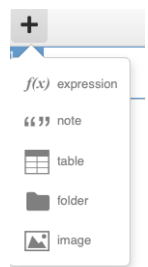


These are step-by-step instructions to create the Cartesian to polar plotting interactive.

A common way to sketch a polar curve is to sketch a function $f(x)$ in the Cartesian plane and then consider how this curve “translates” (not the use of quotation marks here!) to a curve $r = f(\theta)$ in a polar plane.

This interactive allows a demonstration of this, and the different settings allow a few different features to be observed.

When creating this, it is good practice to use the “add note” feature to add some descriptive text for the user.



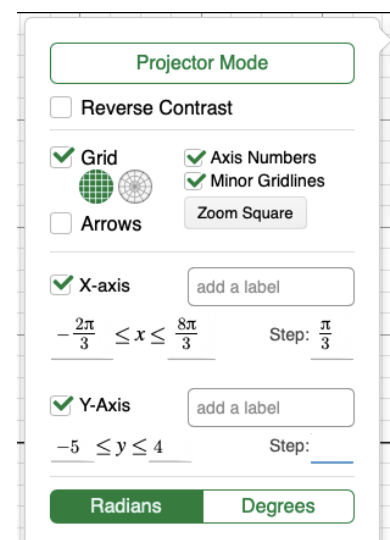
Step 1:

The first thing to do is to set the axes to something useful.

Using the menu contained within the wrench icon to the top right of the Desmos screen set the x -axis to go from $-\frac{2\pi}{3}$ to $\frac{8\pi}{3}$ in steps of $\frac{\pi}{3}$. Also set the y -

axis to go from -5 to 4 . Of course you don't have to use these limits, but they seem to work quite well.

Once you have set your axes limits you need to check that radians are selected as the angle unit and then click the “Zoom Square” button to ensure your



axes are equal-aspect.

Step 2:

Using function notation enter a function $f(x)$ that will be plotted in the Cartesian plane and then plotted as the polar function $r = f(\theta)$. For example, type $f(x) = 2\cos(x)$.

Step 3:

We now want to set up some variables that will define the range of the parameter θ in our polar function. Typing $l = 0$, $u = 2\pi$ (pressing return in between) will set some values that we can use for the lower and upper bounds of the parameter range. We will want to plot some discrete points within this range, so we create another parameter g which will be the spacing between this points. We have found that $g = \frac{\pi}{6}$ works quite well.

Step 4:

Create a sequence of points between l and u with a spacing of g . Ensure that you name this sequence something so that it can be referred to later on.

Step 5:

So that we can plot the cartesian function over just the parameter range defined in Step 3 we plot it as a parametric function $(t, f(t))$ and set t to vary between l and u .

Step 6:

Create a function $A(t)$ that enables referencing a point on this parametric curve using just one argument.

Step 7:

Using similar commands to Step 5 plot the points on the cartesian curve corresponding to the list of discrete parameter values set up in Step 4.

Step 8:

Again using function notation, plot the polar curve $r(\theta) = f(\theta)$.

Step 9:

Set up a function $B(t)$ to plot points on the polar curve.

As we are working within a cartesian plane we will need to use the mappings $x = r \cos(\theta)$ and $y = r \sin(\theta)$.

Step 10:

Using the function $B(t)$ plot the points corresponding to the sequence of points set up in Step 4.

Step 11:

Create a variable p that will determine which functionality is shown in the interactive. The slider created needs to range from 1 to 3 in steps of 1. These settings will correspond to the following behaviour:

- $p = 1$: Selects a point on the two graphs and allows another slider to animate the mapping between the two.
- $p = 2$: Displays equivalence of length on the Cartesian plot and the Polar plot.
- $p = 3$: Displays the correspondence between points on the two graphs.

Step 12:

Setup a slider m to select a point on the graphs. This slider should again have increments of 1 but this time the lower limit should be zero and the upper limit is the number of items in the sequence created in Step 4. The desmos function `length` will be useful here.

Step 13:

In this step we create the functionality corresponding to $p = 1$.

Firstly we need to create a slider n which will move a point from the Cartesian graph to the corresponding point on the polar graph. The limits of this slider should be 0 and 1. Not entering a step enables a smooth transition from 0 to 1.

The object-oriented programming style syntax $A(t) . x$ selects the x -coordinate attribute of the function $A(t)$ (i.e the x -coordinate of a point on the Cartesian graph). Similarly $B(t) . x$ selects the x -coordinate attribute of the function $B(t)$ (i.e. the x -coordinate of a point on the Polar graph).

Consider the function

$$(1 - n)A(t) . x + nB(t) . x$$

When $n = 0$ this function returns the x -coordinate of a point on the Cartesian graph, and when $n = 1$ this function returns the x -coordinate of a point on the Polar graph. For values of n between 0 and 1 this function returns an x -coordinate somewhere between the two extreme values.

Doing something similar but for the y -coordinates create a parametric plot that moves a point from the Cartesian graph to the Polar graph as the slider for n is varied.

We want to animate the movement of each point on the Cartesian graph to the corresponding point on the Polar graph. To do this we can use list indexing of the form $P[m]$ where P is a list to extract a given entry of our sequence of points created in Step 4.

Using the parametric form and functions of the form discussed above animate the movement of a point from one graph to another.

HINT: To only show this when $p = 1$ you can enclose one of the coordinates of your parametric plot between $\{p=1:\dots\}$.

Step 14:

In this step we create the functionality corresponding to $p = 2$.

Thinking again parametrically, plot lines for a given entry of the sequence created in Step 4 which show the following:

- Cartesian Plot: The perpendicular line from the point on the x -axis to the point on the graph.
- Polar Plot: The line from the pole to the point on the polar plot.

Again you want to restrict these to only show when $p = 2$.

Step 15

In this step we create the functionality corresponding to $p = 2$.

Thinking again parametrically, plot a line from a point on the Cartesian graph to the corresponding point on the Polar graph. Style this line how you wish.

As before you will want to restrict these to only show when $p = 2$.

Additions / Modifications

1. It could be nice to have the polar grid show as well. Unfortunately you can't have two grids side by side in Desmos (without manually creating the axes and doing transformations anyway), but you could manually add the lines of a polar grid. Grouping these into a folder will allow you to toggle them on or off all together.
2. Instead of the line connecting the two points for $p = 3$ you could just highlight the points on the two graphs.