Circle Geometry - Summary Notes and Questions

Equation of a circle

$(x-a)^2+(y-b)^2=r^2$ is considered the general form of a circle which has centre (a,b) and radius r

Question: Complete the table below

Equation	Centre	Radius
$x^2+y^2=9$	$(0,0)$	3
$x^2+y^2=25$	$(0,0)$	5
$x^2+y^2=k^2$	$(0,0)$	k
$(x-2)^2+(y-4)^2=16$	$(2,4)$	4
$(x+3)^2+(y-1)^2=169$	$(-3,1)$	13
$(x-2)^2+(y+2)^2=9$	$(2,-2)$	3
$x^2+(y-3)^2=17$	$(0,3)$	$\sqrt{17}$

When the equation isn't given in the canonical form you can complete the square on the x terms and complete the square on the y terms to find the centre and the radius.

Example:

$$x^2-6x+y^2-8y+9=0$$

$$\Rightarrow \quad (x-3)^2-9+(y-4)^2-16+9=0$$

$$\Rightarrow \quad (x-3)^2+(y-4)^2=16$$

and so the centre and radius of the circle x^2-6x+y^2-8y+9 are $(3,4)$ and 4 respectively.

Question: Find the centre and radius of the circles given below:

A. $x^2-4x+y^2-6y-12=0$ $\quad (2,3), 5$

B. $x^2 + x + y^2 - 3y + 2 = 0.$ $\left(-\frac{1}{2}, \frac{3}{2}\right), \frac{1}{\sqrt{2}}$

C. $x^2 - x + y^2 - 10y + \frac{1}{4} = 0$ $\left(\frac{1}{2}, 5\right), 5$

Equation of the tangent to a circle at a given point

To find the equation of the tangent to a circle at a given point follow these steps:

1. Find the gradient of the line segment joining the centre of the circle to the point (on the circle) through which that tangent passes.
2. Find the negative reciprocal of this, call this m. This will be the gradient of the tangent.
3. Substitute the point given into the equation $y = mx + c$ to find the y-intercept c of the tangent.
4. Write the equation of the tangent in whatever form you are asked to.

Questions:

A. Find the equation of the tangent to the circle $(x - 2)^2 + (y - 2)^2 = 25$ at the point $(6,5)$ by following the steps given below:

   1. Find the gradient of the line segment joining the centre of the circle to the point (on the circle) through which that tangent passes.

$$\frac{3}{4}$$

2. Find the negative reciprocal of this, call this m. This will be the gradient of the tangent.

$$-\frac{4}{3}$$

3. Substitute the point given into the equation $y = mx + c$ to find the y-intercept c of the tangent.

$$c = 13$$

4. Write the equation of the tangent in the form $y = mx + c$

$$y = -\frac{4}{3}x + 13$$

B. Find the equation of the tangent to the circle $(x + 2)^2 + (y - 5)^2 = 25$ at the point $(2,8)$ by following the steps given below:

1. Find the gradient of the line segment joining the centre of the circle to the point (on the circle) through which that tangent passes.

$$\frac{3}{4}$$

2. Find the negative reciprocal of this, call this m. This will be the gradient of the tangent.

$$-\frac{4}{3}$$

3. Substitute the point given into the equation $y = mx + c$ to find the y-intercept c of the tangent.

$$c = \frac{32}{3}$$

4. Write the equation of the tangent in the form $ax + by = c$

$$4x + 3y = 32$$

C. Find the equation of the tangent to the circle $(x-1)^2 + (y-1)^2 = 100$ at the point $(7,9)$. Give your answer in the form $ax + by + c = 0$.

$$6x + 8y - 114 = 0$$

Equation of a circle where two points are given at ends of the diameter

If AB is a diameter of a circle then the equation of the circle can then be found. The centre, C, is the midpoint of the line segment AB and the radius is $r = |CB| = |AC|$.

Questions: For each of the pairs of points given below, please find the centre and radius of the circle with diameter AB. Give the equation of the circle in the canonical form.

A. $A(2,4)$ and $B(6,2)$

Centre = $(4,3)$
Radius = $\sqrt{5}$
Equation of circle: $(x-4)^2+(y-3)^2=5$

B. $A(2,4)$ and $B(8,-4)$

Centre = $(5,0)$
Radius = 5
Equation of circle: $(x-4)^2+y^2=25$

C. $A(-2,3)$ and $B(5,-2)$

Centre = $\left(\frac{3}{2},\frac{1}{2}\right)$

Radius = $\sqrt{\frac{37}{5}}$

Equation of circle: $\left(x-\frac{3}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{37}{2}$

<u>Equation of a Circle passing through 3 points</u>

To find the equation of the circle passing through three points A, B and C follow these steps:

1. Find the gradient of the line segment AB and the midpoint of AB.
2. Use this to find the perpendicular bisector of AB and call this line l_1.
3. Find the gradient of the line segment BC and the midpoint of BC.
4. Use this to find the perpendicular bisector of BC and call this line l_2.
5. Find the point of intersection of l_1 and l_2. This is the centre of your circle, call in D.
6. Find the distance between D and any of the three given points - this is the radius of the circle.
7. Using **5.** and **6.** write down the equation of the circle.

Questions:

A. Find the equation of circle passing through $A(6,1)$, $B(5,6)$ and $C(-3,2)$, fully justifying each step.

1. Find the gradient of the line segment AB and the midpoint of AB.

$m = -7$, midpoint = $\left(\frac{11}{2}, \frac{5}{2}\right)$

2. Use this to find the perpendicular bisector of AB and call this line l_1.

$x - 7y = -12$

3. Find the gradient of the line segment BC and the midpoint of BC.

$m = \frac{1}{2}$, midpoint = $(1,4)$

4. Use this to find the perpendicular bisector of BC and call this line l_2.

$-2x - y = -6$

5. Find the point of intersection of l_1 and l_2. This is the centre of your circle, call in D.

$(2,2)$

6. Find the distance between D and any of the three given points - this is the radius of the circle.

$r = 5$

7. Using **5.** and **6.** write down the equation of the circle.

$(x - 2)^2 + (y - 2)^2 = 25$

B. Find the equation of circle passing through $A(15,4)$, $B(8, -13)$ and $C(-2,11)$, fully justifying each step.

1. Find the gradient of the line segment AB and the midpoint of AB.

$m = \frac{17}{7}$, midpoint = $\left(\frac{23}{2}, -\frac{9}{2}\right)$

2. Use this to find the perpendicular bisector of AB and call this line l_1.

$7x + 17y = 4$

3. Find the gradient of the line segment BC and the midpoint of BC.

$m = -\dfrac{12}{5}$, midpoint = $(3, -1)$

4. Use this to find the perpendicular bisector of BC and call this line l_2.

$-5x + 12y = -27$

5. Find the point of intersection of l_1 and l_2. This is the centre of your circle, call in D.

$(3, -1)$

6. Find the distance between D and any of the three given points - this is the radius of the circle.

$r = 13$

7. Using **5.** and **6.** write down the equation of the circle.

$(x - 3)^2 + (y + 1)^2 = 169$

Miscellaneous Circle Questions

You can expect to have to use all your straight line coordinate geometry knowledge when answering circle geometry questions, along with methods for solving simultaneous equations and knowledge of circle theorems.

Here are a couple of more mixed questions for you to try.

A.

1. Show that the triangle ABC where $A(10,2)$, $B(2,10)$ and $C(-2, -10)$ is right angled at the point A.

Gradient of $AB = -1$
Gradient of $AC = 1$
Hence the lines AB and AC are perpendicular, meaning the angle at A is $90°$.

2. Explain how this fact allows you to easily work out the equation of the circle passing through the points A, B and C.

Using a circle theorem BC must be a diameter of the circle passing through the three points, so the midpoint of BC is the centre of the circle.

3. Find the equation of this circle.

$$x^2 + y^2 = 104$$

B. The line segment AB, where the points A and B are $A(a, -4)$ and $B(2,6)$ respectively, is the diameter of a circle.
Given that the point $(-2, -2)$ also lies on the line segment AB, and that the equation of the circle is $(x-b)^2 + (y-c)^2 = d$ find the values of the rational numbers a, b, c and d.

$$a = -3$$
$$b = -\frac{1}{2}$$
$$c = 1$$
$$d = \frac{125}{4}$$

C. The line $3x + 4y = 45$ is a tangent to the circle c at the point $(7,6)$. The line passing through the points $(0,5)$ and $(3,9)$ is also tangent to c at $(0,5)$. Find the equation of the circle.
Hint: Consider perpendicular lines.

$$(x - 4)^2 + (y - 2)^2 = 25$$