Name: …………………………………………………………

1) Complete the gaps in the below definition:

[5]

"For the _____________ function f, given by $f: x \mapsto f(x)$, ______ $\in\mathbb{R}$, then providing that $f(0)$, _____, $f''(0), \dots, f^{(r)}(0)$ all have finite values,

$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}\ x^2 + \cdots + \frac{f^{(r)}(0)}{r!}\ x^r + \cdots$

This is known as _____________ theorem, and the power series is known as the

____________ __________.

2) Using Maclaurin's theorem and differentiation show that

$$e^{2x} = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \cdots + \frac{2^r x^r}{r!} + \cdots$$

[6]

3) Given that x is so small that terms in x^3 and higher powers of x may be neglected, show that

$$11\sin(x) - 6\cos(x) + 5 = A + Bx + Cx^2$$

And state the values of the constants A, B and C.

[10]

4) a) Write down, in ascending powers of x the series for $\cos(4x)$ up to, and including the term in x^6

[4]

b) Hence, or otherwise, show that the first three non-zero terms in the expansion of $\sin^2(2x)$ are $4x^2 - \frac{16}{3}x^4 + \frac{128}{45}x^6$

[6]

5) Find, from first principles (i.e. by differentiation) the series expansion for $\ln(x)$ in ascending powers of $(x-1)$ up to and including the term in $(x-1)^3$.

[8]

6) For the differential equation

$$(1+2x)\frac{dy}{dx} = x + 4y^2$$

find the series solution for $y(x)$ in ascending powers of x, up to and including the term in x^3. You may assume that $y(0) = \frac{1}{2}$.

[10]

7) Using a series approximation to the integrand (up to, and including, the x^6 term estimate the value of $\int_0^{0.6} e^{-x^2}\ dx$, giving your answer to 3 decimal places.

[8]

8) a) If $y = \ln(\cos(x))$, prove that

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2}\frac{dy}{dx} = 0$$

[7]

b) Hence, or otherwise, obtain the Maclaurin expansion of $y(x)$ in terms of x up to and including the term in x^4.

[8]

c) Using $x = \frac{\pi}{4}$ show that $\ln(2) \approx \frac{\pi^2}{16}\left(1 + \frac{\pi^2}{96}\right)$

[5]

9) The function $y(x)$ satisfies $y'' = xy$. By differentiating n times, show that, for ≥ 1, $y^{(n+2)}(0) = ny^{(n-1)}(0)$. Given that $y(0) = 1$ and $y'(0) = 0$, find the first four non-vanishing terms of the Maclaurin series for y.

[10]

Topic	Score	Confident?	Need to review?	Need a lot more work?	WORK DONE
Knowing the definition	/5				
Showing a Maclaurin expansion.	/6				
Approximating functions for small x.	/10				
Using standard expansion to derive other expansions.	/10				
Deriving a Taylor expansion	/8				
Finding a series solution to a differential equation.	/10				
Using series to approximate definite integrals	/8				
Deriving a Maclaurin expansion by differentiating.	/20				
A taster undergraduate question	/10				

Chapter 2 summary	/87	%	A=80%	B=70%	C=60%	D=50%	E=40%	<40% U