Do Now

1) Find the exponential form and modulus argument form of the complex number $2 + 3\mathrm{i}$.

2) Find $\left(-\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}\right)^2$

3) Find the square root of $-28 + 96\mathrm{i}$

nth Roots of a Complex Number

Let z be a complex number and w also be a complex number. Then if n is a positive integer the equation $z^n = w$ has distinct solutions.

For the examples we are going to make use of the fact that

$$z = r\left(\cos(\theta) + \mathrm{i}\sin(\theta)\right)$$
$$=$$

Example

a) Solve the equation $z^3 = 1$

b) Represent the solutions from **(a)** on an Argand diagram.

c) Show that the cube roots of 1 can be written as 1, ω and ω^2 where $1 + \omega + \omega^2 = 0$

Example

Solve the equation $z^4 = 8\sqrt{3} + 8\mathrm{i}$

Exercise

1) Find the fifth roots of unity.

2) Solve $z^3 - 27 = 0$

3) Solve the equation $z^4 - \frac{75}{2} - \frac{81\sqrt{3}}{2}\mathrm{i} = 3.$

4) Solve the equation $1 + z + z^2 + z^3 + z^4 + z^5 = 0$.

Hint: The sum of a geometric series will be useful here.