6 **a)** Find the first three terms, in ascending powers of x, of the binomial expansion of $\dfrac{1}{\sqrt[3]{8+2x}}$

[3 marks]

$$\frac{1}{\sqrt[3]{8+2x}} = (8+2x)^{-\frac{1}{3}} = \left(8\left(1+\tfrac{1}{4}x\right)\right)^{-\frac{1}{3}} = \frac{1}{2}\left(1+\tfrac{1}{4}x\right)^{-\frac{1}{3}}$$

$$= \frac{1}{2}\left(1 + \left(-\tfrac{1}{3}\right)\left(\tfrac{1}{4}x\right) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}\left(\tfrac{1}{4}x\right)^2 + \ldots\right)$$

$$= \frac{1}{2}\left(1 - \frac{1}{12}x + \frac{1}{72}x^2 + \ldots\right)$$

$$= \frac{1}{2} - \frac{1}{24}x + \frac{1}{144}x^2 + \ldots$$

b) Hence, find the expansion of $\dfrac{1}{\sqrt[3]{8-2x^2}}$

[2 marks]

$$\frac{1}{\sqrt[3]{8-2x^2}} = \frac{1}{2} - \frac{1}{24}\left(-x^2\right) + \frac{1}{144}\left(-x^2\right)^2 + \ldots$$

$$= \frac{1}{2} + \frac{1}{24}x^2 + \frac{1}{144}x^4 + \ldots$$

c) Millie uses the first three terms of the expansion found in **(b)** to find an approximation to the the integral $\displaystyle\int_0^{\frac{1}{2}} \frac{2}{\sqrt[3]{8-2x^2}}\,\mathrm{d}x.$

Evaluate this approximation.

[3 marks]

$$\int_0^{\frac{1}{2}} \frac{2}{\sqrt[3]{8-2x^2}} dx = 2\int_0^{\frac{1}{2}} \frac{1}{\sqrt[3]{8-2x^2}} dx \approx 2\int_0^{\frac{1}{2}} \left(\frac{1}{2} + \frac{1}{24}x^2 + \frac{1}{144}x^4\right)dx$$

$$= \int_0^{\frac{1}{2}} \left(1 + \frac{1}{12}x^2 + \frac{1}{72}x^4\right)dx$$

$$= \left[x + \frac{1}{36}x^3 + \frac{1}{360}x^5\right]_0^{\frac{1}{2}} = \left(\frac{5801}{11520}\right) - (0) = \frac{5801}{11520} \approx 0.504$$