

AQA A-Level Further Maths 2022 Paper 3

Statistics

Do not turn over the page until instructed to do so.

This assessment is out of 50 marks and you will be given 60 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: / 50

Solutions



- 1 The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{7}, & 2 \leq x \leq 9 \\ 0, & \text{otherwise} \end{cases}$$

Find $P(X \geq 4)$.

$\frac{5}{7}$

$\frac{4}{7}$

1

0

[1 mark]

- 2 The continuous random variable Y has probability density function

$$f(y) = \begin{cases} \frac{6}{125}y(5 - y), & 0 \leq y \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

Find $P(Y = 3)$.

$\frac{6}{125}$

$\frac{36}{125}$

0

$\frac{81}{125}$

[1 mark]

- 3 Let X be a discrete random variable which follows a discrete uniform distribution, taking values $1, 2, 3, \dots, n$.

a) Prove that $E(X) = \frac{n+1}{2}$

[3 marks]

$$\begin{aligned}
 E(X) &= \sum_{i=1}^n \frac{x_i}{n} \\
 &= \frac{1+2+\dots+n}{n} \\
 &= \frac{\frac{n}{2}(n+1)}{n} \\
 &= \frac{n+1}{2}
 \end{aligned}$$

- 4 The sales of a particular brand of mobile phone at two shops which are in the same group follow independent Poisson distributions with parameters $\lambda_1 = 4$ and $\lambda_2 = 5.5$ respectively.

a) Why is a Poisson distribution a suitable model here?

[1 mark]

Events (sales) occur in a given interval (day) independently at random

b) Find the probability that the first shop sells more than 6 phones.

[2 marks]

Let $X_1 \sim P_0(4)$ and $X_2 \sim P_0(5.5)$

$$\begin{aligned} P(X_1 > 6) &= 1 - P(X_1 \leq 6) \\ &= 1 - 0.8893 \\ &= 0.1107 \end{aligned}$$

c) (i) The area manager is interested in the combined sales of the two stores. What distribution could he use to model the same across the two stores? State the mean and standard deviation of this distribution.

[3 marks]

Let $Z = \lambda_1 + \lambda_2$, then
 $Z \sim P_o(\lambda_1 + \lambda_2)$
so $Z \sim P_o(9.5)$

Mean = 9.5

standard deviation = $\sqrt{9.5}$

- (ii) Find the probability that the combined sales on a given day were between 5 and 11 units. *inclusive*

[2 marks]

$$\begin{aligned}P(5 \leq Z \leq 11) &= P(Z \leq 11) - P(Z \leq 4) \\&= 0.751989 - 0.040266 \\&= 0.7117 \text{ to 4dp}\end{aligned}$$

- 5 The probability distribution, N , for the number of times someone visits a supermarket in 4 days is modelled by the probability distribution given below.

n	0	1	2	3	4	5	6
$P(N=n)$	k	$3k$	$4k$	$3k$	$2k$	k	k

Calculate the mean and the variance.

[4 marks]

Finding k

$$k + 3k + 4k + 3k + 2k + k + k = 1 \Rightarrow 15k = 1 \text{ so } k = \frac{1}{15}$$

Mence the distribution is

n	0	1	2	3	4	5	6
$P(N=n)$	$\frac{1}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{1}{15}$

$$E(N) = \sum x P(N=x)$$

$$= 0 \times \frac{1}{15} + 1 \times \frac{3}{15} + 2 \times \frac{4}{15} + 3 \times \frac{3}{15} + 4 \times \frac{2}{15} + 5 \times \frac{1}{15} + 6 \times \frac{1}{15}$$

$$= \frac{13}{5}$$

$$= 2.6$$

$$\text{Var}(N) = E(N^2) - (E(N))^2$$

$$= \sum x^2 P(N=x) - \left(\frac{13}{5}\right)^2$$

$$= \left(0 \times \frac{1}{15} + 1^2 \times \frac{3}{15} + 2^2 \times \frac{4}{15} + 3^2 \times \frac{3}{15} + 4^2 \times \frac{2}{15} + 5^2 \times \frac{1}{15} + 6^2 \times \frac{1}{15}\right) - \left(\frac{13}{5}\right)^2$$

$$\approx 2.507$$

- 6 A random variable, X , has probability density function

$$f(x) = \begin{cases} -kx(x-3) & 0 \leq x < 3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

- a) Find k .

[2 marks]

$$\begin{aligned} & \int_0^3 -kx(x-3) dx = 1 \\ \Rightarrow & \left[-\frac{k}{3}x^3 + \frac{3}{2}kx^2 \right]_0^3 = 1 \\ \Rightarrow & \frac{9}{2}k = 1 \\ \text{so } & k = \frac{2}{9} \end{aligned}$$

- b) Find the median of the distribution.

[3 marks]

$$\begin{aligned} & \int_0^M -\frac{2}{9}x(x-3) dx = \frac{1}{2} \\ \Rightarrow & \frac{M^2}{3} - \frac{2M^3}{27} = \frac{1}{2} \\ \text{so } & M = \frac{3}{2}, \frac{3-3\sqrt{3}}{2} \text{ or } \frac{3+3\sqrt{3}}{2} \end{aligned}$$

So the median = $\frac{3}{2}$ since others are not in the domain of the distribution

c) Find the mean of the distribution

[3 marks]

$$E(X) = \int_0^3 -\frac{2}{9}x^2(x-3) dx$$

$$= 1.5$$

d) Find the cumulative distribution function.

[3 marks]

$$\int_0^t -\frac{2}{9}x^2(x-3) dx$$

$$= -\frac{1}{18}(t-4)t^3$$

7

- a) Describe the Yate's Correction and explain when it is used.
[2 marks]

Used for a 2x2 contingency table.

The calculation of the χ^2 statistic is replaced

$$\text{by } \chi_{\text{Yates}}^2 = \sum \frac{(|O_i - E_i| - 0.5)^2}{E_i}$$

- b) In a school the catering manager is interested in whether pupils' break time choices are associated with the year they are in. He collects the following table of data.

O_i	Bacon Cob	Pizza	Totals
Year 7	11	24	35
Year 11	19	17	36
Totals	30	41	71

Perform a hypothesis test for association at a 5% significance level.

[6 marks]

E_i calculations

E_i	Bacon	Pizza
Yr 7	14.79	20.21
Yr 11	15.21	20.79

χ^2 contributions

	Bacon Cob	Pizza
Yr 7	0.731351442	0.581352075
Yr 11	0.710361242	0.5202703347

$$\text{Test Statistic} = 2.4978$$

$$D_{df} = (2-1) \times (2-1) \\ = 1$$

The critical value at 5% is 3.841

Since $2.4978 < 3.841$ there is not enough evidence to reject the null hypothesis and we conclude that there isn't any evidence of association between year group and breaktime snack choice.

- 8 A can of blood orange Italian soda is claimed to contain 16.2 g of sugar. A sample of size 10 is taken which has the following sample statistics.

$$\bar{x} = 15.9 \text{ g}$$

$$S^2 = 3.4 \text{ g}^2$$

- a) Gillian suggests taking the sample from one sealed pack of 12 cans. Explain why this may be problematic.

[1 mark]

These cans were likely produced together in the factory and so wouldn't constitute a random sample.

- b) Use the data to construct a 95 % confidence interval for the mean amount of sugar per can.

[3 marks]

$$\bar{x} \pm K \frac{s}{\sqrt{n}}$$

Hence $K = 2.262, s^2 = 3.4$

$$\bar{x} \pm 2.262 \times \sqrt{3.4}$$

$15.48290781, 16.31709219$

Hence confidence interval is
 $(15.5, 16.3)$

- b) What hypothesis test could be performed in this situation? You must state your assumptions on the distribution of the sample that are required.

[2 marks]

Since the population variance is unknown and the sample size is small we can use a t-test.
The sample must come from a normal distribution

- c) Perform this test and decide if there is reason to doubt the manufacturers claim

[5 marks]

$$H_0: \mu_0 = 16.2$$

$$H_1: \mu_0 \neq 16.2$$

t-test with $10 - 1 = 9$ degrees of freedom

$$\begin{aligned} t &= \frac{15 - 9 - 16.2}{\sqrt{\frac{34}{10}}} \\ &= -0.5145 \end{aligned}$$

At the 10% level for a two-tailed test (so 5% in each tail), the critical value for 9 degrees of freedom is -2.262.

Since $-0.5145 > -2.262$ there is insufficient evidence to suggest things are not as the manufacturer claims

- 9 Let X be a random variable with an exponential distribution which has parameter λ .

b) If $\lambda = \frac{1}{3}$ for the exponential distribution described above, calculate

i) $P(X < 1)$

$$\text{Let } F(x) = 1 - e^{-\frac{x}{3}} \quad [1 \text{ mark}]$$

$$\begin{aligned} \text{Then } P(X < 1) &= F(1) \\ &= 1 - e^{-\frac{1}{3}} \\ &\approx 0.2835 \end{aligned}$$

ii) $P(X > 3)$

[1 marks]

$$\begin{aligned} P(X > 3) &= 1 - P(X < 3) \\ &= 1 - (1 - e^{-\frac{3}{3}}) \\ &= e^{-1} \\ &\approx 0.3679 \end{aligned}$$

- c) Four x values are taken from the distribution $Y \sim \text{Exp}\left(\frac{1}{2}\right)$.

Find the probability that 3 of them are less than 1 and the remaining value is bigger than 1.

[3 marks]

Let Z be the RV : the number of values out of 4 that are less than one?

Then

$$Z \sim B(4, 0.3934) \quad \text{From P}$$

Then

$$P(Z=3) = 0.147 \quad (\text{why not check})$$