AQA A-Level Further Maths 2022 Paper 3 Statistics

Do nut turn over the page until instructed to do so.

This assessment is out of 50 marks and you will be given 60 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: / 50

1 The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{7}, & 2 \leq x \leq 9 \\ 0, & \text{otherwise} \end{cases}$$

Find $\mathrm{P}(X \geq 4)$.

$\frac{5}{7}$ $\frac{4}{7}$ 1 0

[1 mark]

2 The continuous random variable Y has probability density function

$$f(y) = \begin{cases} \frac{6}{125}y(5-y), & 0 \leq y \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

Find $\mathrm{P}(Y = 3)$.

$\frac{6}{125}$ $\frac{36}{125}$ 0 $\frac{81}{125}$

[1 mark]

3 Let X be a discrete random variable which follows a discrete uniform distribution, taking values $1,2,3,\ldots,n$.

a) Prove that $\mathrm{E}(X) = \dfrac{n+1}{2}$

[3 marks]

4 The sales of a particular brand of mobile phone at two shops which are in the same group follow independent Poisson distributions with parameters $\lambda_1 = 4$ and $\lambda_2 = 5.5$ respectively.

a) Why is a Poisson distribution a suitable model here?

[1 mark]

b) Find the probability that the first shop sells more than 6 phones.

[2 marks]

c) **(i)** The area manager in turreted in the combined sales of the two stores. What distribution could he use to model the same across the two stores? State the mean and standard deviation of this distribution.

[3 marks]

(ii) Find the probability that the combined sales on a given day were between 5 and 11 units.

[2 marks]

5 The probability distribution, N, for the number of times someone visits a supermarket in 4 days is modelled by the probability distribution given below.

n	0	1	2	3	4	5	6
$P(N = n)$	k	$3k$	$4k$	$3k$	$2k$	k	k

Calculate the mean and the variance.

[4 marks]

6 A random variable, X, has probability density function

$$f(x) = \begin{cases} -kx(x-3) & 0 \leq x < 3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

a) Find k.

[2 marks]

b) Find the median of the distribution.

[3 marks]

c) Find the mean of the distribution

[3 marks]

d) Find the cumulative distribution function.

[3 marks]

7

a) Describe the Yate's Correction and explain when it is used.

[2 marks]

b) In a school the catering manager is interested in whether pupils' break time choices are associated with the year they are in. He collects the following table of data.

O_i	Bacon Cob	Pizza	Totals
Year 7	11	24	35
Year 11	19	17	36
Totals	30	41	71

Perform a hypothesis test for association at a 5% significance level.

[6 marks]

8 A can of blood orange Italian soda is claimed to contain $16.2\ \text{g}$ of sugar. A sample of size 10 is taken which has the following sample statistics.

$$\bar{x} = 15.9\ \text{g}$$
$$S = 3.4\ \text{g}^2$$

a) Gillian suggests taking the sample from one sealed pack of 12 cans. Explain why this may be problematic.

[1 mark]

b) Use the data to construct a $95\ \%$ confidence interval for the mean amount of sugar per can.

[3 marks]

b) What hypothesis test could be performed in this situation? You must state your assumptions on the distribution of the sample that are required.

[2 marks]

c) Perform this test and decided if there is reason to doubt the manufacturers claim

[5 marks]

9 Let X be a random variable with an exponential distribution which has parameter λ.

b) If $\lambda = \frac{1}{3}$ for the exponential distribution described above, calculate

i) $P(X < 1)$

[1 mark]

ii) $P(X > 3)$

[1 marks]

c) Four x values are taken from the distribution $Y \sim \text{Exp}\left(\frac{1}{2}\right)$.

Find the probability that 3 of them are less than $1$and the r remaining value is bigger than 1.

[3 marks]