AQA A-Level Further Maths 2022 Paper 3 Mechanics

Do nut turn over the page until instructed to do so.

This assessment is out of 50 marks and you will be given 60 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: / 50

Solutions

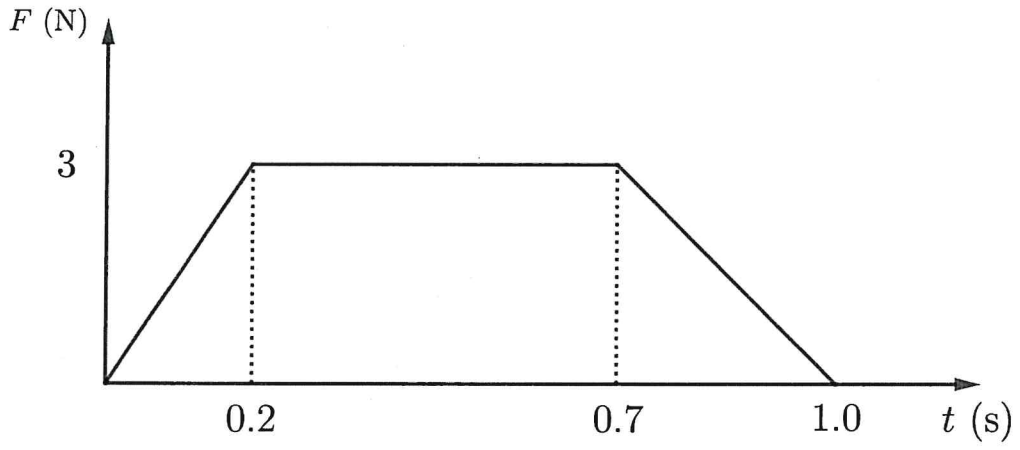
1 A rigid, uniform rod, AB, of length 5 m has mass 4 kg. A particle of mass 3 kg is placed 2 m from B, and a particle of mass 5 kg is placed at B to form a composite body.

Find the distance of the centre of mass of the composite body from A.

$\frac{11}{5}$ m $\quad$ $\frac{11}{3}$ m $\quad$ $\frac{17}{4}$ m $\quad$ $\frac{4}{3}$ m

[1 mark]

2 The graph below shows how a force, F, varies with time over a one second period.



Find the magnitude of the impulse of F during this second.

2.25 Ns $\quad$ 3 Ns $\quad$ 1.5 Ns $\quad$ 1.95 Ns

[1 mark]

3 A goods train of mass 150 tonnes is travelling on a straight horizontal track.

It has a maximum speed of 19 ms^{-1} when its engine is working at 950 kW.

Find the magnitude of the resistive force acting on the train when it is travelling at its maximum speed.

[3 marks]

Maximum speed implies that the tractive force = resistive force.

So, using $P = Fv$

$950000 = F \times 19$

$\Rightarrow F = 50{,}000\ N$

4 A particle moves on a horizontal plane in which the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular. At time t, the particles position vector is

$$\mathbf{r} = 5\cos(2t)\mathbf{i} - 5\sin(2t)\mathbf{j}$$

a) Prove that the particle is moving in a circle, centre the origin and state its radius.

[2 marks]

$$|\underset{\sim}{r}| = \sqrt{5^2\cos^2(2t) + 5^2\sin^2(2t)}$$

$$= \sqrt{25(\cos^2(2t) + \sin^2(2t))}$$

$$= 5$$

Since $|\underset{\sim}{r}|$ is a constant the particle is moving in a circle

b) Show that the acceleration can be written as $\mathbf{a} = k\mathbf{r}$ for some k you should find.

[3 marks]

$$\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = -10\sin(2t)\underset{\sim}{i} - 10\cos(2t)\underset{\sim}{j}$$

then $\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt}$

$$= -20\cos(2t)\underset{\sim}{i} + 20\sin(2t)\underset{\sim}{j}$$

$$= -4(5\cos(2t)\underset{\sim}{i} - 5\sin(2t)\underset{\sim}{j})$$

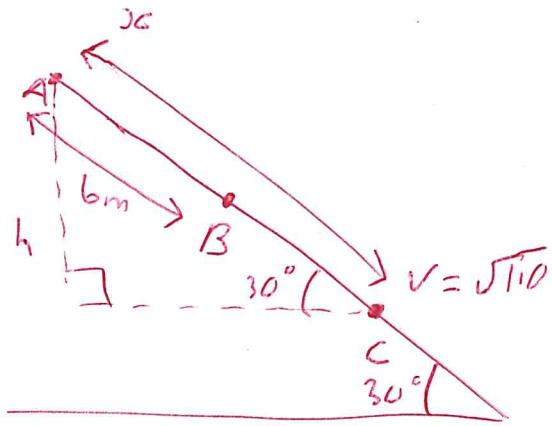
$$= -4\underset{\sim}{r}$$

5 A particle of mass 3 kg is placed on a smooth plane inclined at 30° to the horizontal.

It is released from rest at a point A and moves in a straight line down the plane. It moves past the point B which is 6 m down the plane from A. It subsequently passes the point C which is further down the plane. When at C the speed of the particle is $\sqrt{110}$ ms^{-1}.

Find the distance BC.

[6 marks]



$\sin(30) = \frac{h}{x}$

$\Rightarrow h = x\sin(30) = \frac{x}{2}$

Assuming no air resistance, since the plane is smooth the gain in kinetic energy is equal to the loss of potential energy

$mg\frac{x}{2} = \frac{1}{2}m(\sqrt{110})^2$

$\Rightarrow \frac{9x}{2} = 55$

$\Rightarrow x = \frac{55 \times 2}{9.8}$

$= \frac{550}{49}$

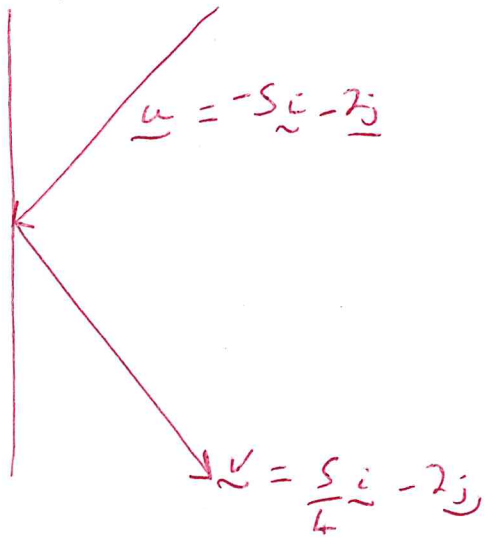
Hence $|BC| = \frac{550}{49} - 6 = \frac{256}{49} \approx 5.22\text{m}$

6 A small smooth ball of mass of 1.5 kg is moving in the XY plane and collides with a smooth fixed vertical wall containing the $y-$axis.

The velocity of the ball just before impact is $\mathbf{u} = -5\mathbf{i} - 2\mathbf{j}$ and just after it is $\mathbf{v} = \frac{5}{4}\mathbf{i} - 2\mathbf{j}$.

a) Find the speed of the ball before and after impact.

[2 marks]



$|\underset{\sim}{u}| = \sqrt{(-5)^2 + (-2)^2}$

$= \sqrt{29}$

$\approx 5.39 \text{ ms}^{-1}$

$|\underset{\sim}{v}| = \sqrt{\left(\frac{5}{4}\right)^2 + (-2)^2}$

$= \sqrt{\frac{89}{16}}$

$\approx 2.35 \text{ ms}^{-1}$

b) Find the loss of kinetic energy as a result of the impact.

[2 marks]

Loss of KE $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

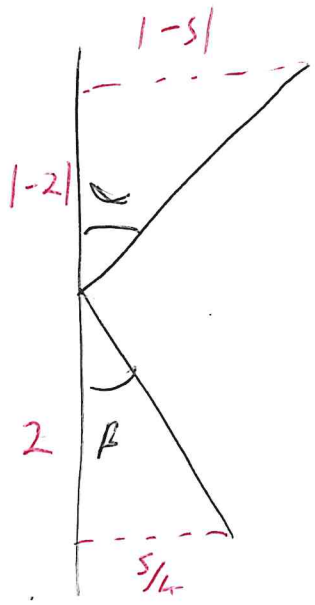
$= \frac{1}{2} \times 1.5 \times 29 - \frac{1}{2} \times 1.5 \times \frac{89}{16}$

$= \frac{1125}{64}$

$\approx 17.6 \text{ J}$

c) Find the angle of deflection of the ball.

[3 marks]



$\tan(\alpha) = \frac{s}{2}$

$\Rightarrow \alpha = 68.19859051$

$\tan(\beta) = \frac{s/4}{2} = s/8$

$\Rightarrow \beta = 32.00538321$

So angle of deflection $= \alpha + \beta$

$= 100.2039737$

$\approx 100.2°$

For use $\cos\theta = \frac{\underline{u} \cdot \underline{v}}{|\underline{u}||\underline{v}|} = \frac{-9/4}{\sqrt{29}\sqrt{\frac{89}{16}}} \Rightarrow \theta \approx 100.2°$

d) Show that the coefficient of restitution between the ball and the wall is $\frac{1}{4}$.

[2 marks]

$V_\perp = e u_\perp$

$\Rightarrow \frac{s}{4} = e \times s$

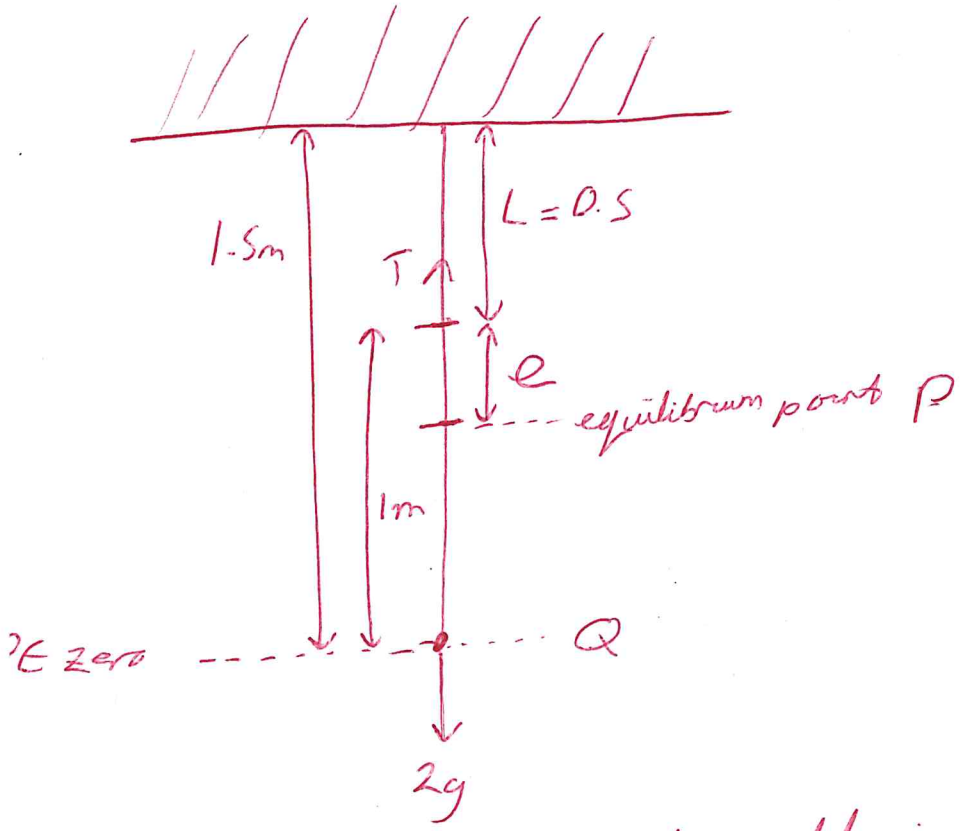
$\Rightarrow e = \frac{1}{4}$

7 A particle of mass 2 kg is attached to one end of a light elastic string of natural length 0.5 metres. and modulus of elasticity 50 N. The other end of the wall is fixed to a point O on a ceiling beam.

a) The particle hangs in equilibrium at the point P, below O. Find the extension in the string at this point.

[2 marks]

Diagram for (a) and (b)



Let e be the extension at equilibrium

At P, as in equilibrium

$$T = 2g,$$

but by Hooke's law $T = \frac{\lambda e}{L}$, so

$$2g = \frac{50e}{0.5}$$

$$\Rightarrow e = 0.196 \text{ m}$$

b) The particle is then pulled down to a point Q which is 1.5 m below O.

The particle is then released. By considering energy, find the velocity of the particle when it passes back through the equilibrium point.

[4 marks]

At P: $KE = \frac{1}{2}mv^2$

$PE = mgh = 2 \times 9.8 \times \frac{201}{250} = \frac{9849}{625}$

$EPE = \frac{\lambda e^2}{2l} = \frac{50 \times 0.196^2}{2 \times 0.5} = 1.9208$

At Q: $KE = 0$

$PE = 0$

$EPE = \frac{50 \times 1^2}{2 \times 0.5} = 50$

So by energy conservation,

$$50 = v^2 + \frac{9849}{625} + 1.9208$$

$$\Rightarrow v^2 = 32.3208$$

$$\Rightarrow v = \sqrt{32.3208}$$

$$\approx 5.69 \text{ ms}^{-1}$$

8 Sophie is investigating how the speed, v, of waves on a string depends on the mass, m, of the string, the length, l, of the string and the tension, t, in the string.

She conjectures a relationship of the form

$$v = km^{\alpha}l^{\beta}t^{\gamma},$$

where k is a dimensionless constant.

Determine the values of α, β and γ.

[5 marks]

$[v] = [m^{\alpha}][L^{\beta}][t^{\gamma}]$

$\Rightarrow LT^{-1} = M^{\alpha}L^{\beta}(MLT^{-2})^{\gamma}$

So

L| $1 = \beta + \gamma$ (1)

T| $-1 = -2\gamma$ (2)

M| $0 = \alpha + \gamma$ (3)

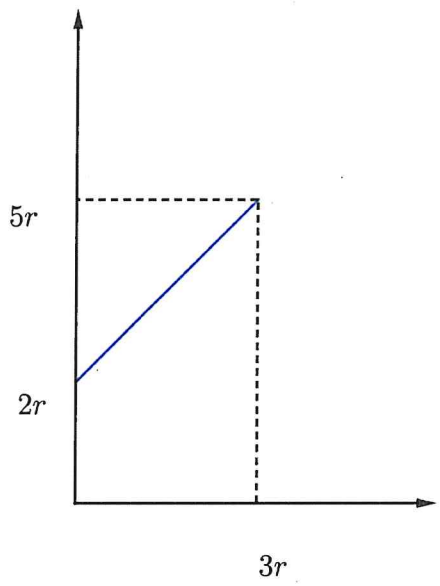
(2) $\Rightarrow \gamma = \frac{1}{2}$

$\Rightarrow \alpha = -\frac{1}{2}$

$\Rightarrow \beta = \frac{1}{2}$

Hence $v = km^{-1/2}L^{1/2}t^{1/2}$

9 The region bounded by the line $y = x + 2r$, the $y-$axis, the $x-$axis and the line $x = 3r$ is shown below.



This region is rotated around the $x-$axis to form the frustum of a uniform solid cone.

a) Show that the volume of the frustum is $39\pi r^3$ units cubed.

[2 marks]

$$V = \pi \int_0^{3r} y^2 \, dx$$

$$= \pi \int_0^{3r} (x+2r)^2 \, dx$$

$$= \pi \int_0^{3r} x^2 + 4rx + 4r^2 \, dx$$

$$= \pi \left[\frac{x^3}{3} + \frac{4rx^2}{2} + 4r^2 x \right]_0^{3r}$$

$$= 39\pi r^3$$

b) Find the distance of the centre of mass of the frustum from O.

[3 marks]

Consider $\pi \int_0^{3r} x y^2 \, dx$

$$= \pi \int_0^{3r} x (x + 2r)^2 \, dx$$

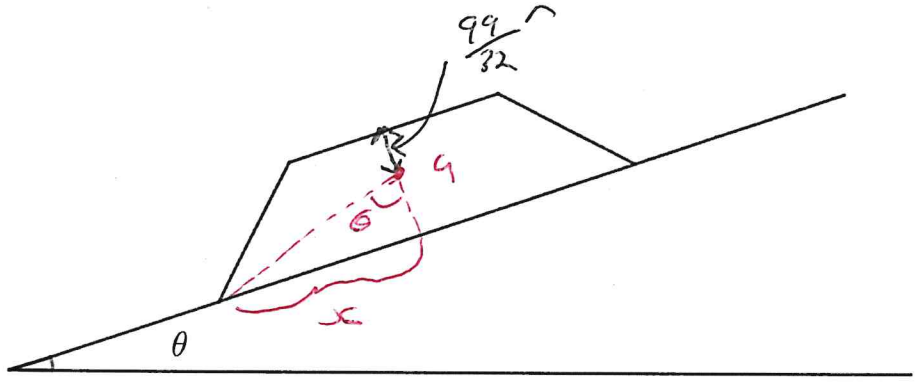
$$= \frac{297\pi r^4}{4}$$

Hence, centre of mass is at a distance of

$$\frac{\frac{297\pi r^4}{4}}{39\pi r^3} = \frac{99}{52} r$$

away from the origin, along the x-axis

c) This frustum is now placed on a rough plane inclined at an angle θ to the horizontal as shown.



If the plane is sufficiently rough to prevent sliding, find the maximum value of θ for the frustum to remain in equilibrium without toppling.

[3 marks]

then $\tan(\theta) = \dfrac{5r}{3r - \frac{99r}{32}}$

$= \dfrac{5}{57/32}$

$\Rightarrow \tan(\theta) = \dfrac{160}{57}$

$\Rightarrow \theta = \tan^{-1}\left(\dfrac{160}{57}\right)$

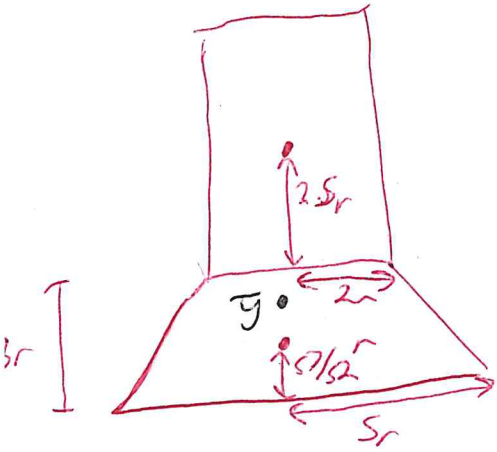
$\approx 77.63^\circ$

d) A cylinder (made of the same material as the frustum) is now placed on top of the smaller face of the frustum. The cylinder has radius $2r$ and height $5r$.

This composite body is now placed on a plane inclined at an angle α to the horizontal such that the larger face of the frustum is in contact with the plane.

Suppose again that the surface is sufficiently rough so as to prevent sliding. Find the maximum value of inclination of the plane for the composite body to not topple.

[5 marks]



Volume of cylinder $= 20\pi r^3$

Volume of frustum $= 39\pi r^3$

Then,

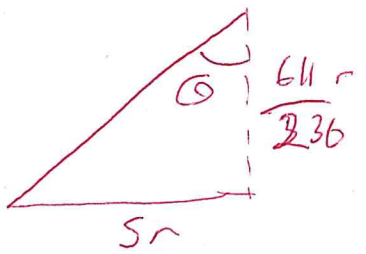
$$(20+39)\pi r^3 \bar{y} = 39\pi r^3 \times \frac{57}{52} r + 20\pi r^3 \times 5.5r$$

$$\Rightarrow \quad 59\bar{y} = \frac{611}{4} r$$

$$\Rightarrow \quad \bar{y} = \frac{611}{256} r$$

$$\approx 2.59r$$

Let the angle of inclination be θ°, then



$$\tan(\theta) = \frac{5r}{\frac{611r}{236}}$$

$$= \frac{1180}{611}$$

$$\Rightarrow \theta = \arctan\left(\frac{1180}{611}\right)$$

$$\approx 62.6^\circ$$

e) Explain why the surfaces that are sufficiently rough to prevent sliding in **(c)** and **(d)** are not necessarily the same surface.

[1 mark]

The point of slipping is not necessarily the same.