AQA A-Level Further Maths 2022 Paper 3 Discrete

Do nut turn over the page until instructed to do so.

This assessment is out of 50 marks and you will be given 60 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: **/ 50**

Solutions

1 A connected planar graph has x vertices and $2x - 2$ edges.

Find the number of faces of the planar graph in terms of x.

x $\quad x-2 \quad x+2 \quad x-4$

[1 mark]

2 Consider the binary operation $a \star b = a + b$ on the set $\mathbb{R}$.

Which of the following statements is true.

The operation $a \star b$ is associative, not commutative and does not have and inverse ☐

The operation $a \star b$ is associative, commutative and does not have an identity. ☐

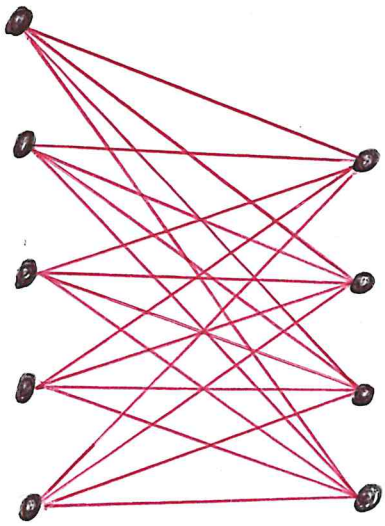
The operation $a \star b$ is asociatve, not commutative and has an identity. ☐

The operation $a \star b$ is associative, commutative and has an identity. ☑

[1 mark]

3 **a)** Draw the graph $K_{5,4}$

[2 marks]

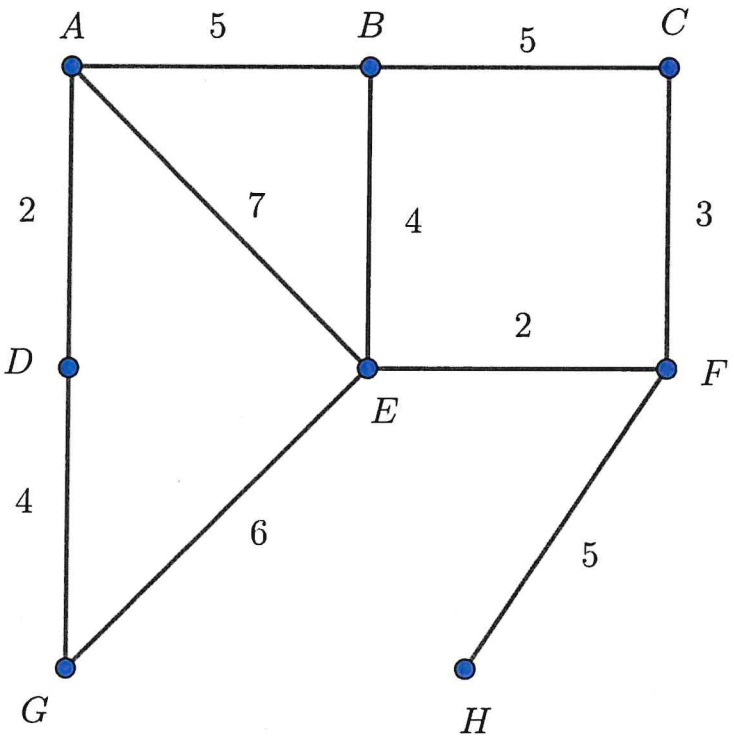


b) Explain how you know that $K_{5,4}$ is non-planar.

[2 marks]

$K_{3,3}$ is a subgraph of $K_{5,4}$ and so by Kuratowski's Theorem is non-planar.

4 The network below represents the distances, in kilometres, between 8 towns on a cycle network.



a) Identify the odd nodes.

[1 mark]

Node	Nº edges
A	3
B	3
C	2
D	2
E	4
F	3
G	2
H	1

So the odd nodes are A, B, F and H

b) Frida wishes to start at A and cycle along each path at least once, returning to A.
Find the length of the shortest route that allows this to happen.
[4 marks]

Consider the pairings

AB	5
AF	9
AH	14
BF	6
BH	11
FH	5

Now consider

AB	FH	5+5 = 10
AF	BH	9+11 = 20
AH	BF	14+6 = 20

Combination giving shortest distance is therefore AB and FH.

So repeating arcs AB and FH.

Total distance is 43+10 = 53km

c) Frida changes her plan so that she starts at A and ends in a cafe at F.

What difference does this make to the total number of kilometres covered if she still wants to cover each path at least once?

[2 marks]

A and F remain odd so need to pair up BH.
Shortest distance between BH is 11, so total distance is now
43 + 11 = 55 km.

5 Consider the set $G = \{1, i, -1, -i\}$ under multiplication.

a) Construct the Cayley table for G under the binary operation of multiplication.

[2 marks]

$\times$	1	i	-1	$-i$
1	1	i	-1	$-i$
i	i	-1	$-i$	1
-1	-1	$-i$	1	i
$-i$	$-i$	1	i	-1

b) Explain why $(G, \times)$ is a group.

[3 marks]

$\times$ is a closed binary operation which is associative.
There is an identity element, 1
Each $a \in G$ has an inverse $a^{-1} \in G$.

c) What are the generators of the group. State their period.

[2 marks]

i is period 4

$-i$ is period 4

d) What are the proper subgroups of $(G, \times)$ under multiplication.

[1 mark]

$\{+1\}$ and $\{-1, 1\}$

6 Consider the activity table shown below.

Activity	Predecessors	Duration
A	-	2
B	-	3
C	A	4
D	A,B	2
E	D	5

a) Complete, in the space on the next page, an activity network for the activities listed below, showing the earliest start and latest finish times.

Use the table format

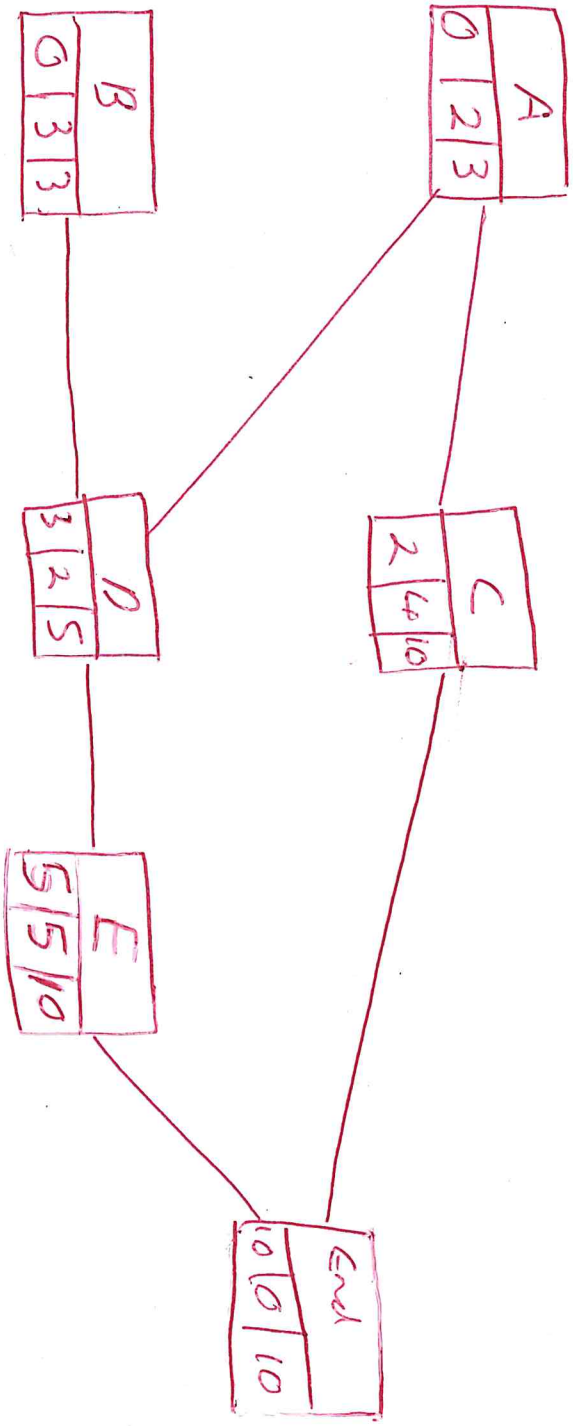
Activity		
Earliest Start Time	Duration	Latest Finish Time

[4 marks]

b) Identify the non-critical activities.

[2 marks]

A, C


A
0 2 3
B
6 3 3
C
2 4 10
D
3 2 5
E
5 5 10
End
10 10 10

7 Consider the game represented by the pay-off matrix below.

	A	B	C
A	-1	2	1
B	3	1	2

a) Set up and then apply one iteration of the simplex method to the pay off matrix above.

[4 marks]

Prob (A) $= p_1$

Prob (B) $= p_2$

Add 2 (or more) to all entries.

	A	B	C
A	1	4	3
B	5	3	4

Then payoffs are $p_1 + 5p_2$, $4p_1 + 3p_2$ and $3p_1 + 4p_2$

We want to maximise

$P = v - 2$ such that

$v \leq p_1 + 5p_2$

$v \leq 4p_1 + 3p_2$

$v \leq 3p_1 + 4p_2$

$p_1 + p_2 \leq 1$

$p_1, p_2 \geq 0$

Add slack variables.

Maximise $P = v - 2$

subject to

$v - p_1 - 5p_2 + s_1 = 0$

$v - 4p_1 - 3p_2 + s_2 = 0$

$v - 3p_1 - 4p_2 + s_3 = 0$

$p_1 + p_2 + s_4 = 1$

Simplex tableau

P	V	P_1	P_2	S_1	S_2	S_3	S_4	Value
1	-1	0	0	0	0	0	0	-2
0	1	-1	-5	1	0	0	0	0
0	1	-4	-3	0	1	0	0	6
0	1	-3	-4	0	0	1	0	0
0	0	1	1	0	0	0	1	1

Pivot row or column indicated or pivot circled

After 1 iteration

P	V	P_1	P_2	S_1	S_2	S_3	S_4	Value
1	0	-4	-3	0	1	0	0	-2
0	0	3	-2	1	-1	0	0	0
0	1	-4	-3	0	1	0	0	0
0	0	1	-1	0	-1	1	0	0
0	0	1	1	0	0	0	1	1

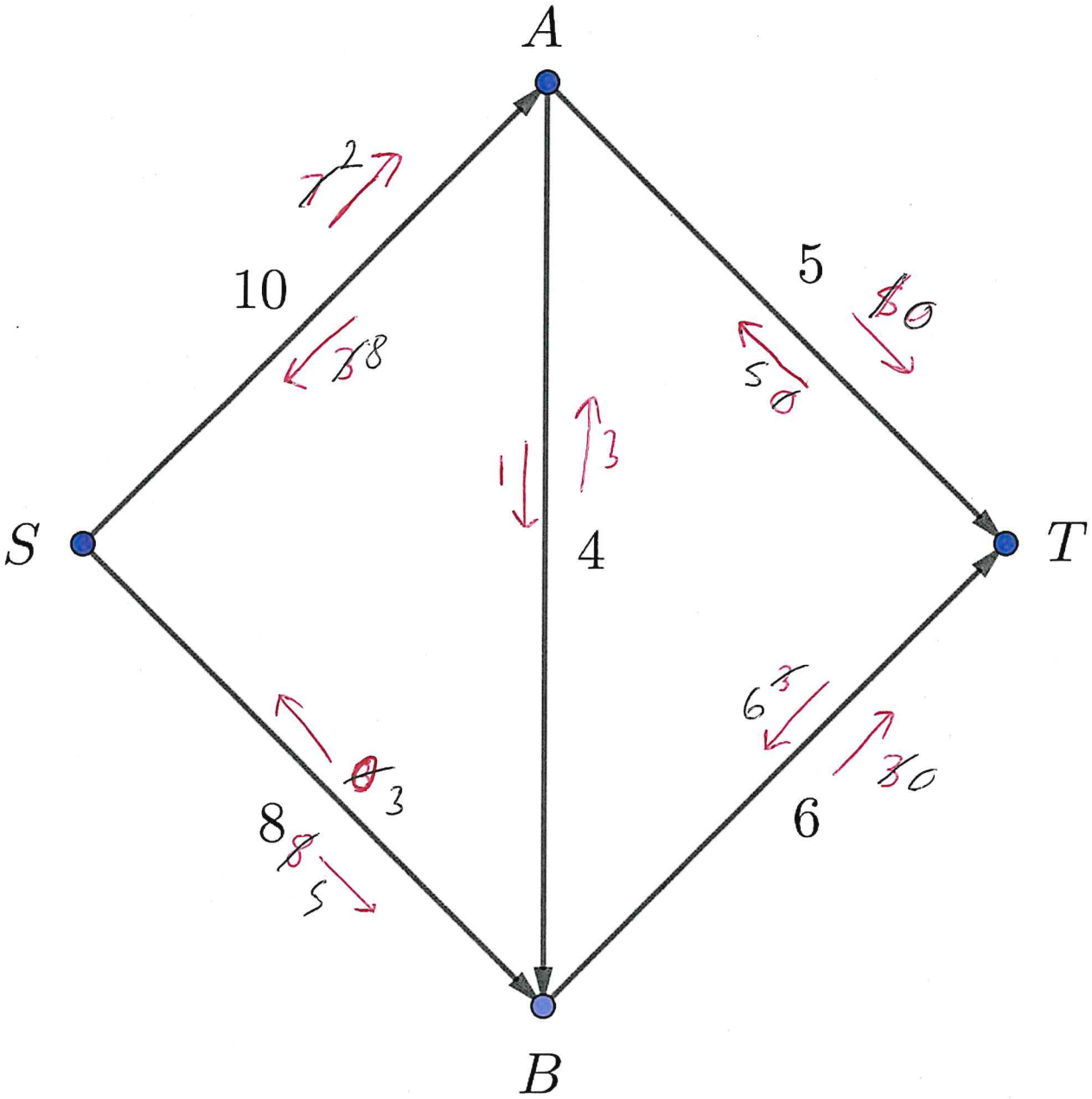
b) How do you know that this is not an optimal solution?

[2 marks]

There are still negative numbers in the objective row with positive numbers in that column.

8 Starting with an initial flow of 3 along the network $SABT$, find a maximal flow.

[5 marks]

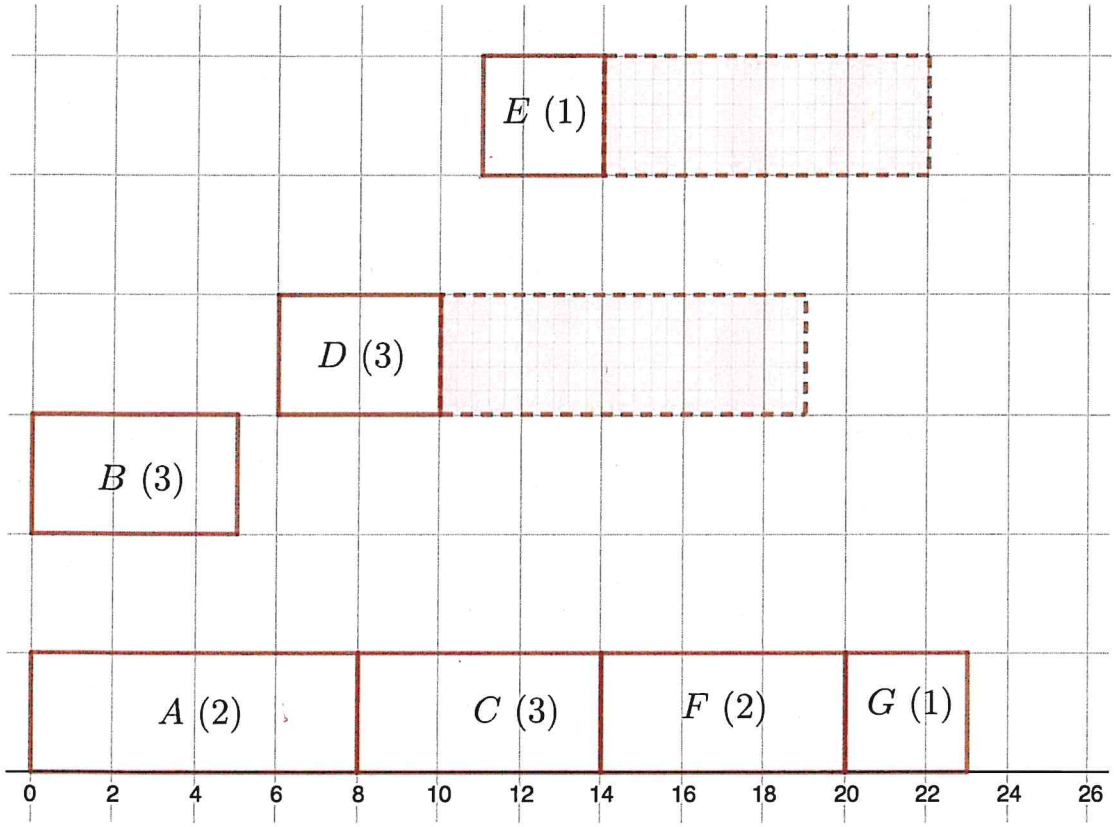


SAT flow augmenting path, potential for 5

SBT flow augmenting path of value 3

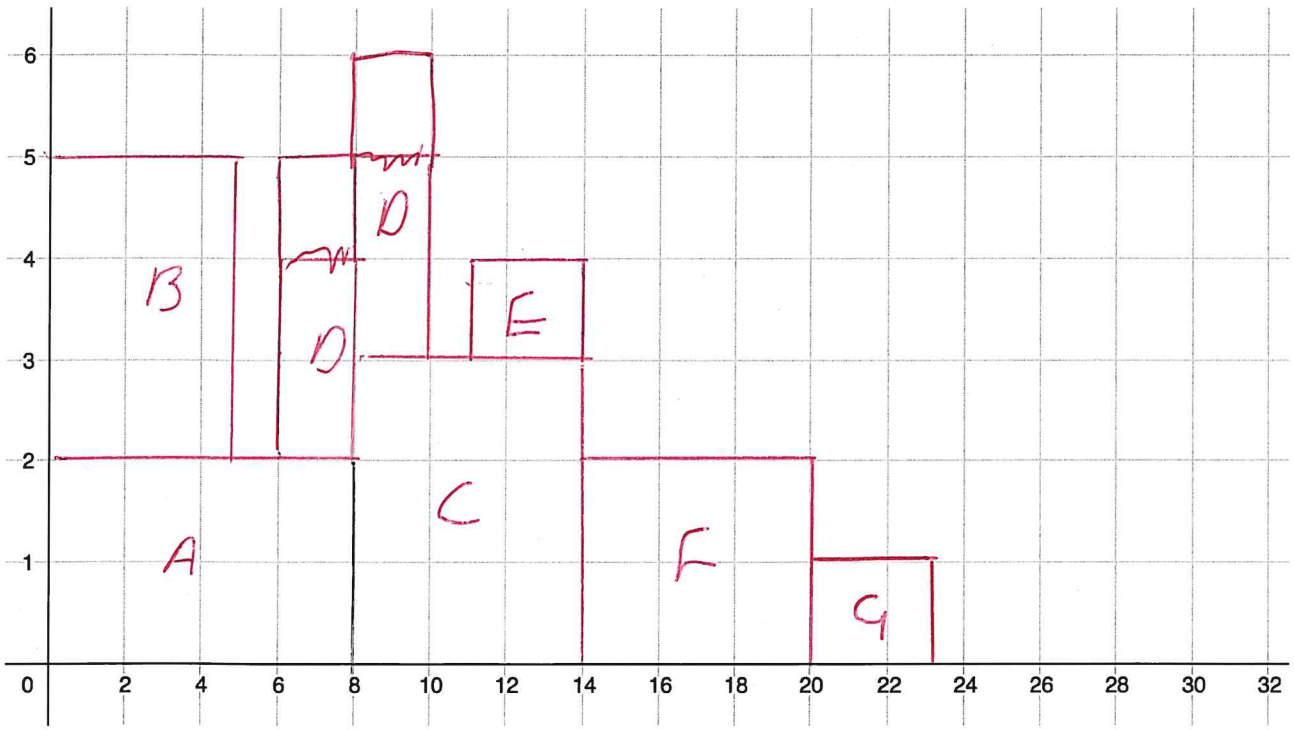
Maximal flow = 11

9 The Gantt chart below shows a project lasting 23 days. The value in the brackets gives the number of workers required for each activity.



a) Draw a resource histogram assuming that each activity starts as early as possible.

[4 marks]



b) Show that you can complete the project within 30 days by delaying some activities and using only ~~30~~ 25 workers.

[3 marks]

10 Abi and Jack play a zero sun game where they choose to play either a King, Queen or Jack card.

The payoff matrix for Abi is shown below:

		Jack		
		K	**Q**	**J**
Abi	**K**	5	-3	2
	Q	2	3	4
	J	3	-2	-1

a) Show that there does not exist a stable solution to this zero sum game.

[3 marks]

Row minima : -3, 2, -2

$\max(\text{row minima}) = 2$

Column maxima 5, 3, 4

$\min(\text{col maxima}) = 3$

Since $\max(\text{row minima}) \neq \min(\text{col maxima})$

there is not a stable solution

b) What is Abi's play safe strategy and why?

[1 mark]

To play Queen as her worst outcome is she does is to win 2.

c) By identifying dominated strategies reduce the payoff matrix as far as possible, explaining when you are finished.

[2 marks]

Column Q dominates Column J

		Jack	
		K	Q
Abi	K	5	-3
	Q	2	3
	J	3	-2

No other dominated strategies exist