# AQA A-Level Further Maths 2022 Paper 3 Discrete 

Do nut turn over the page until instructed to do so.
This assessment is out of 50 marks and you will be given 60 minutes.

When you are asked to by your teacher write your full name below

## Name:

## Total Marks: <br> / 50

1 A connected planar graph has $x$ vertices and $2 x-2$ edges.

Find the number of faces of the planar graph in terms of $x$.

$$
\begin{array}{cccc}
x & x-2 & x+2 & x-4
\end{array}
$$

[1 mark]

2 Consider the binary operation $a \star b=a+b$ on the set $\mathbb{R}$.
Which of the following statements is true.

The operation $a \star b$ is associative, not commutative and does not have and inverse

The operation $a \star b$ is associative, commutative and does not have an identity.

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3 a) Draw the graph $K_{5,4}$
[2 marks]
b) Explain how you know that $K_{5,4}$ is non-planar.

4 The network below represents the distances, in kilometres, between 8 towns on a cycle network.

a) Identify the odd nodes.
b) Frida wishes to start at $A$ and cycle along each path at least once, returning to $A$.
Find the length of the shortest route that allows this to happen.
[4 marks]
c) Frida changes her plan so that she starts at $A$ and ends in a cafe at $F$.

What difference does this make to the total number of kilometres covered if she still wants to cover each path at least once?
[2 marks]

5 Consider the set $G=\{1, \mathrm{i},-1,-\mathrm{i}\}$ under multiplication.
a) Construct the Cayley table for $G$ under the binary operation of multiplication.
[2 marks]

| $\times$ | 1 | i | -1 | -i |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |
| i |  |  |  |  |
| -1 |  |  |  |  |
| -i |  |  |  |  |

b) Explain why $(G, \times)$ is a group.
[3 marks]
c) What are the generators of the group. State their period.
[2 marks]
d) What are the proper subgroups of $(G, \times)$ under multiplication.
[1 mark]

6 Consider the activity table shown below.

| Activity | Predecessors | Duration |
| :---: | :---: | :---: |
| A | - | 2 |
| B | - | 3 |
| C | A | 4 |
| D | A,B | 2 |
| E | D | 5 |

a) Complete, in the space on the next page, an activity network for the activities listed below, showing the earliest start and latest finish times.

Use the table format

| Activity |  |  |
| :---: | :---: | :---: |
| Earliest | Duration | Latest <br> Finish <br> Time |
| Start Time |  |  |

[4 marks]
b) Identify the non-critical activities.
[2 marks]

7 Consider the game represented by the pay-off matrix below.

|  | A | B | C |
| :---: | :---: | :---: | :---: |
| $\mathbf{A}$ | -1 | 2 | 1 |
| B | 3 | 1 | 2 |

a) Set up and then apply one iteration of the simplex method to the pay off matrix above.
[4 marks]
b) How do you know that this is not an optimal solution?

8 Starting with an initial flow of 3 along the network $S A B T$, find a maximal flow.
[5 marks]


Maximal flow $=$

9 The Gantt chart below shows a project lasting 23 days. The value in the brackets gives the number of workers required for each activity.

a) Draw a resource histogram assuming that each activity starts as early as possible.
[4 marks]

b) Show that you can complete the project within 30 days by delaying some activities and using only 30 workers.
[3 marks]


10 Abi and Jack play a zero sun game where they choose to play either a King, Queen or Jack card.

The payoff matrix for Abi is shown below:

a) Show that there does not exist a stable solution to this zero sum game.
b) What is Abi's play safe strategy and why?
[1 mark]
c) By identifying dominated strategies reduce the payoff matrix as far as possible, explaining when you are finished.
[2 marks]

